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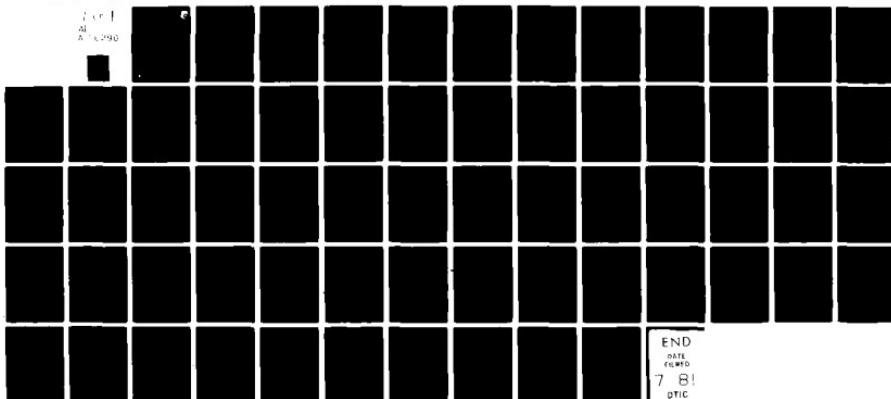
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PREDICTION OF THE MECHANICAL RESPONSE OF  
A HIGH TEMPERATURE SUPERALLOY RENE 95

UNIVERSITY OF CINCINNATI  
CINCINNATI, OHIO

D. C. Stouffer  
L. Papernik  
H. L. Bernstein

September 1980

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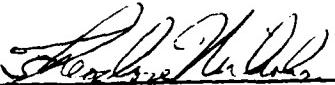
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THEODORE NICHOLAS

Metals Behavior Branch  
Metals and Ceramics Division



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WALTER H. REIMANN, Acting Chief  
Metals Behavior Branch  
Metals and Ceramics Division

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parameters. Second, it is found that creep and stress relaxation can be correlated with isochronous curves; thus, the theory of Rabotnov and Papernik is examined. The model is extended to cyclic histories and a representation is developed specifically for Rene 95. Third, the model developed by Laflen and Stouffer for another super alloy is extended to Rene 95. It is also used to predict reverse plastic flow developed during cyclic histories.

All models were found to reproduce the stress-strain and creep response very well. The models were also used to predict the cyclic behavior and stress relaxation. In general it is found that none of the models can adequately predict phenomena arising from damage such as tertiary creep. The study is concluded with a point-by-point comparison of the models.

## FOREWORD

This Technical Report was prepared by the Department of Aerospace Engineering and Applied Mechanics of the University of Cincinnati. The Engineer is Dr. T. Nicholas. This report describes the research results of the constitutive modeling of René 95 at 650°C (1200°F) and is the completion of Tasks 4.2.1 and 4.2.3 to 4.2.8. of Contract F33615-78-C-5199, "Constitutive Modeling". The research reported was conducted by D. C. Stouffer, L. Papernik and H. L. Bernstein from September 1978 to July 1980. The report was submitted for publication in December 1980.

The authors express appreciation to Dr. T. Nicholas for his suggestions and comments during the project. The authors also express sincere thanks to Joe Conway and Ray Stentz of Mar-Test, Inc., Cincinnati, Ohio, for their cooperation in conducting the experimental program; and, Professor Sol Bodner for reviewing the manuscript prior to publication. The authors thank Vivek Rao and Kulwant Virdi for their efforts in reducing the data and developing the plotter programs to produce the curves displayed in this document.

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## SECTION I INTRODUCTION

The objective of this research is to develop procedures to predict the family of mechanical response characteristics for René 95 at 650°C (1200°F). This is achieved by adapting three constitutive theories available in the literature for René 95; determining the material coefficients from a standard set of tensile and creep experiments; and, then predicting the response of several experiments distinctly different from the standard set of experiments used to find the material constants.

The mechanical response of René 95 at 650°C (1200°F) is typical in some ways of many high temperature alloys in that it exhibits creep, stress relaxation, strain recovery, and both cyclic hardening and softening. However, as discussed in Reference 1, René 95 also exhibits some special response characteristics that must be included in a constitutive model. These characteristics are: (i) the relative rate independence in the stress strain response at the higher strain rates using engineering stress; (ii) that the material is nearly history independent in the secondary creep domain; and (iii) the development of a mean strain during symmetric load control cycling.

Constitutive modeling of metal behavior on the basis of single integral hereditary equations, state variable equations, and also on the basis of a yield surface approximation is available in the literature. For René 95 at elevated temperature, the yield surface methods have several shortcomings. In particular, since inelastic deformations are observed at stresses well below the standard proportional limit, a continuous flow type equation appears to be more consistent. This avoids the necessity of separating the inelastic strain into time independent and time dependent components. Thus, only constitutive

equations that predict the total inelastic strain or total strain as a continuous function of stress are considered.

In this study, specific attention is given to the models of: Bodner and Partom (References 2-5); Laflen and Stouffer (Reference 6), and Rabotnov and Papernik (References 7-8). As reported earlier (Reference 9), all three models are continuous and phenomenologically developed, but emphasize different physical considerations and mathematical representations. All theories are three dimensional and have been successfully used to predict the response of one or more metals in a high temperature environment. Modifications in the theories have been made or proposed as a result of the present study. The predictive capability of each model is evaluated by examining the following points: (1) how well each theory can reproduce the data from which its material constants are evaluated; (2) how well it can predict the response of a totally unrelated set of experiments; (3) how easily the material constants can be determined from the experimental data; and (4) the efficiency of each model when it is used in a numerical algorithm. The paper is concluded with a point by point comparison of the above three models.

## A STATE VARIABLE APPROACH

### SECTION II THE CONSTITUTIVE FORMULATION

The basis of the constitutive equations for small strains proposed by Bodner and Partom (References 2-5) is the separation of the total strain rate tensor  $\dot{\underline{\epsilon}}(t)$ , into elastic (reversible) and inelastic (non-reversible) components

$$\dot{\underline{\epsilon}}(t) = \dot{\underline{\epsilon}}^e(t) + \dot{\underline{\epsilon}}^p(t) \quad (1)$$

that are assumed to be continuous and non-zero for all non-zero values of stress. As a consequence, the equations do not require a yield criterion or loading/unloading conditions. The term  $\dot{\underline{\epsilon}}^e(t)$  is directly related to the time derivative of stress by Hooke's Law for the small strain case. In general  $\dot{\underline{\epsilon}}^p(t)$  is assumed to be of a form similar to the Prandtl-Reuss equations

$$\dot{\underline{\epsilon}}^p(t) = \dot{\underline{\epsilon}}^p(t) = \lambda \hat{\underline{\sigma}}(t) \quad (2)$$

where  $\lambda$  is a scalar material function and  $\dot{\underline{\epsilon}}^p$  and  $\hat{\underline{\sigma}}$  are the deviatoric strain and stress tensors. In general  $\lambda$  is taken as a function of the stress, temperature and state variables, however; since this exercise is for a constant temperature environment, temperature is omitted from the representation. The dependence of  $\lambda$  on stress and the state variables is outlined in the next few paragraphs.

Equation 2 predicts that the response is isotropic and that the plastic strains are incompressible. (These restrictions can be removed as shown in Reference 5). The square of Equation 2 can be rewritten in the form

$$D_2^P = \lambda^2 J_2 \quad (3)$$

where  $D_2^P$  and  $J_2$  are the second invariants of the plastic strain rate and deviatoric stress tensors, respectively. To develop a relationship between plastic strain rate and stress, Bodner and Partom, Reference 2, 3, assumed that some measure of the inelastic strain rate, namely  $D_2^P$ , should have a mathematical form similar to the relationship between the average velocity of mobile dislocations,  $v$ , and the applied stress,  $\sigma$ . Following the work of Vreeland, Reference 10 or Gillman, Reference 11, the dislocation velocity has a stress dependence which can be approximated by

$$v \sim \left[ \frac{\sigma}{A} \right]^n \quad \text{or} \quad v \sim \exp \left( -\frac{B}{\sigma} \right),$$

respectively, where  $A$ ,  $B$ , and  $n$  are constants. Subsequently, Bodner and Partom ultimately evolved a representation in the form

$$D_2^P \sim \exp \left[ - \left( \frac{Z^2}{J_2} \right)^n \right] \quad (4)$$

to obtain the maximum flexibility in the model. Further, experience has shown that  $Z$  is not a constant; but should be interpreted as an internal state variable. Thus, using Equations 4, 3 and 1 gives a specific representation for the plastic strain rate tensor. The model can be written in one dimension as

$$\dot{\epsilon}(t) = \frac{\dot{\sigma}(t)}{E} + \dot{\epsilon}^P(t),$$

where

(5)<sup>(1)</sup>

$$\dot{\epsilon}^p = \frac{2}{\sqrt{3}} D_0 \frac{\sigma(t)}{|\sigma(t)|} \exp \left[ -\frac{1}{2} \left( \frac{n+1}{n} \right) \left( \frac{Z(t)}{\sigma(t)} \right)^{2n} \right].$$

The constant,  $D_0$ , represents the limiting strain rate;  $E$ , the elastic modulus; and  $n$ , a constant controlling the strain rate sensitivity. The term  $\sigma / |\sigma|$  requires that plastic strain rate and stress have the same sign. Note that Equation 5 cannot predict strain recovery since  $\dot{\epsilon}^p = 0$  whenever  $\sigma = 0$ . This deficiency could be relevant for René 95 since a small amount of strain recovery is present at elevated temperatures.

The state variable,  $Z$ , is a macroscopic measure of the hardness or resistance to inelastic flow. The formulation is, at least in part, motivated by the properties of the stored energy of cold work. The evolution equation for  $\dot{Z}$  is therefore assumed to depend on the rate of inelastic working and a hardness recovery term, i.e.

$$\dot{Z} = \left( \frac{\partial Z}{\partial W_p} \right) \dot{W}_p - \dot{Z}_{rec} \quad (6)$$

which is the general form

$$\dot{Z} = f(Z, \sigma)$$

In order to describe both the short time stress-strain response and the long time creep response a specific representation can be written in the form

$$\dot{\frac{Z}{Z_1}} = m \left[ 1 - \frac{Z}{Z_1} \right] \dot{W}_p - A \left\{ \frac{Z - Z_I}{Z_1} \right\}^r \quad (7)$$

---

(1) This factor  $(n + 1)/n$  was introduced at an early stage of the development of the theory for numerical convenience, it has been included in this presentation in order to compare results.

which can be integrated to give

$$\frac{z}{z_1} = z_0 + \int_0^t [1 - \frac{z}{z_1}] dw_p - \int_0^t A \left\{ \frac{z - z_I}{z_1} \right\}^r dt . \quad (8)$$

The parameters  $z_0$ ,  $z_1$ ,  $z_I$ ,  $m$  and  $r$  are all constants. In general, the constants in Equation 7 or 8 are picked so that the integral term is negligible during rapid stress histories (neglecting recovery effects). The constants  $z_0$ ,  $z_1$  and  $m$  are determined from the stress strain data; whereas  $A$ ,  $z_I$  and  $r$  are determined from the creep response.

Finally, for some materials it was found that the strain hardening characteristics required some additional constants. For René 95, this possibility was investigated to improve the predictive capability of the model. Thus, for later use, let us generalize the constant  $m$  to

$$m = m_0 + m_1 \exp(-\alpha w_p), \quad (9)$$

where  $m_0$ ,  $m_1$  and  $\alpha$  are the additional material constants. A catalog of most of the material constants and their physical meaning is given in Reference 12.

It should be noted in using the integrated form for  $Z$ , Equation 8, that  $w_p$  is the relative amount of plastic work from a given state  $z_0$  and is not an absolute value. An interesting point is that secondary creep is the condition for which  $\dot{z} = 0$  which leads to an equation for  $Z = Z(\sigma)$  independent of  $z_0$ , i.e., prior history. According to these equations, the secondary creep rate would be independent of prior history effects.

SECTION III  
PREDICTION OF MATERIAL PROPERTIES

In a recent report, Reference 12, Bodner evaluated the response of René 95 at 650°C (1200°F) from previously published data in Reference 13. In this work  $m$  was fixed as a constant rather than using Equation 8. The values of the constants reported in Reference 12 are:

$$D_0 = 10^4 \text{ sec}^{-1}$$

$\alpha$  = not used

(assumed)

$$Z_I = 1600 \text{ MPa (232 KSI)}$$

$$n = 3.2$$

$$Z_1 = 2200 \text{ MPa (319 KSI)}$$

$$A = 4 \times 10^{-4} \text{ sec}^{-1}$$

$$Z_0 = 1600 \text{ MPa (232 KSI)}$$

$$r = 1.5$$

$$m_0 = 0.4 \text{ MPa}^{-1} (0.058 \text{ KSI}^{-1})$$

$$E = 1.77 \times 10^5 \text{ MPa (2.57} \times 10^4 \text{ KSI)}$$

$$(m_1 = 0)$$

These constants also were used as a starting point to predict the stress-strain and creep data reported in Reference 1. The results are shown in Figures 1, 2, and 3. In general, reasonable agreement is found, however, the predicted response could be improved through the strain range of interest. One shortcoming observed in Figure 1, and also in Figure 1 of Reference 12, is that the predicted shape of the stress-strain curve does not match the data as well as might be desired. Another difficulty is establishing the material constants given above. The system of equations are highly nonlinear with complex coupling. Bodner and his coworkers have proposed a method to find some of the above constants, Reference 12; which they have extended to the above system of equations through a trial and error sequence of numerical exercises. In the next section an attempt is made to improve the model in these areas.

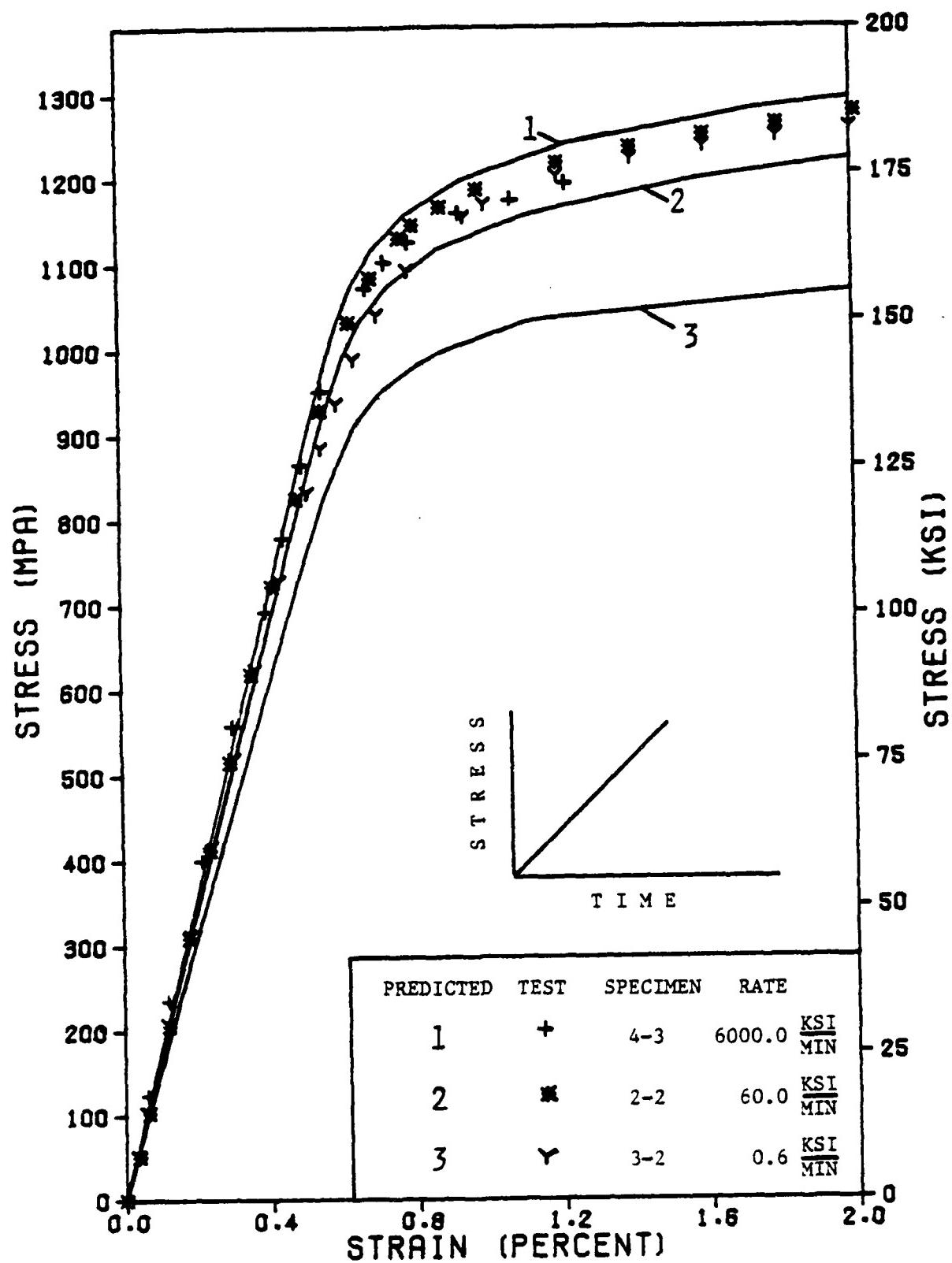


Figure 1. Experimental and predicted response using the state variable theory with Equation (8)

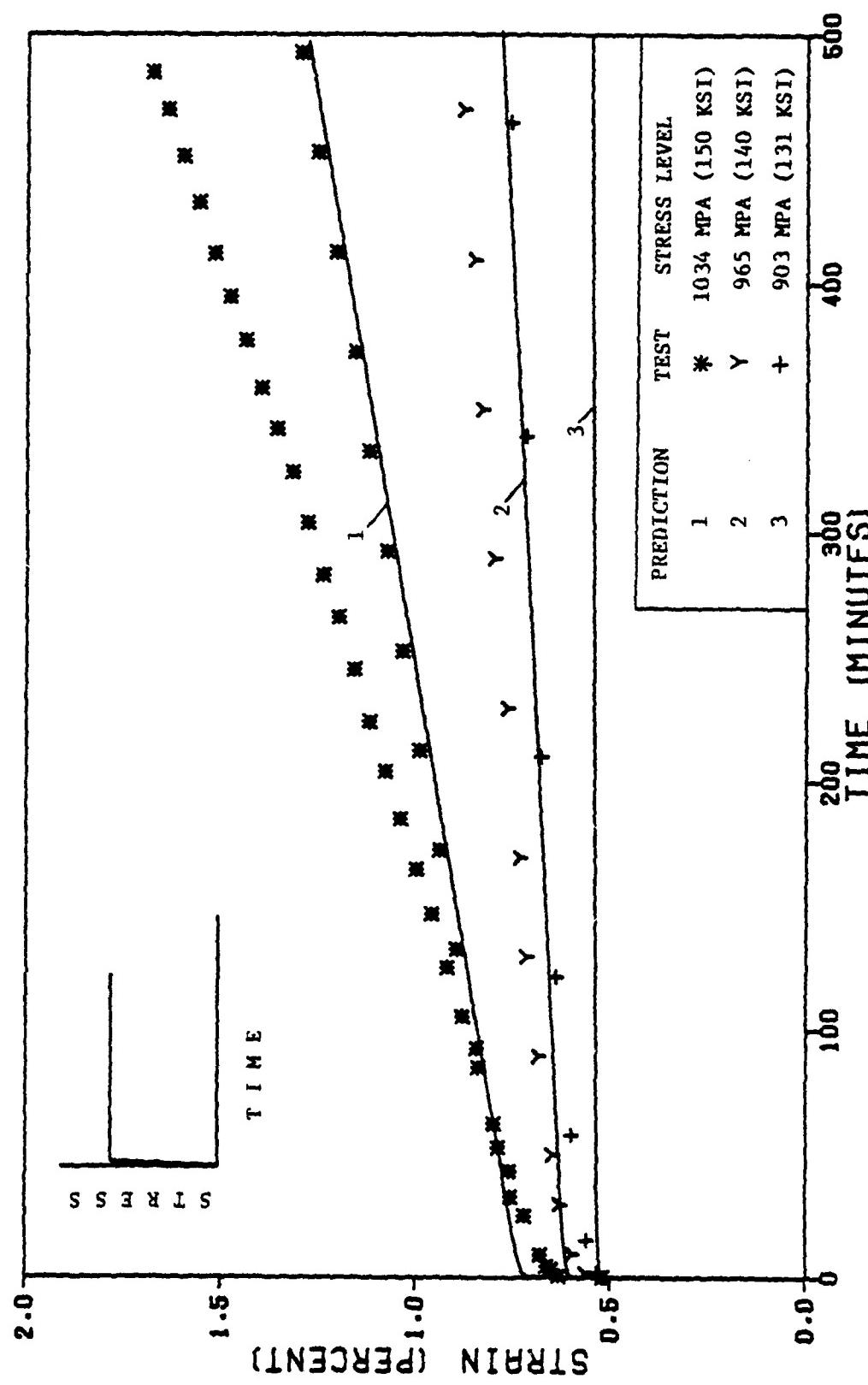


Figure 2. Experimental and predicted long term creep response using the state variable theory

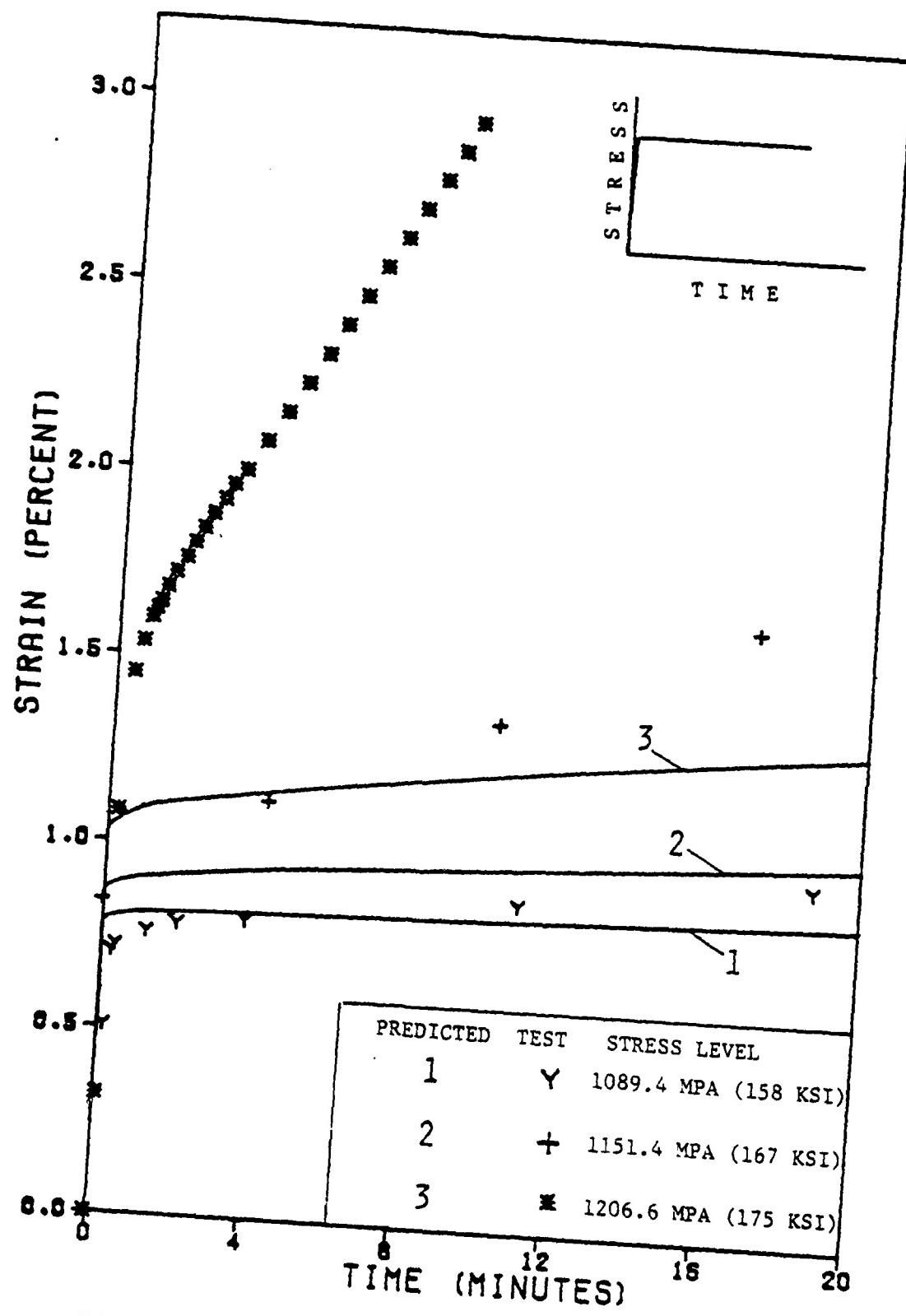


Figure 3. Experimental and predicted short time creep response using the state variable theory

SECTION I<sup>1</sup>

## ANALYSIS OF THE HARDENING PARAMETER

The above system of equations requires the use of 7 or 9 constants that must be determined from the inelastic response of the material. The limiting value of the stress rate,  $D_0$ , does not appear to be critical and is generally assumed to be  $10^4 \text{ sec}^{-1}$ . The elastic modulus is also assumed to be known.

One method to determine the hardening characteristics is to observe that the plastic strain rate history,  $\dot{\epsilon}^P(t)$ , and stress history,  $\sigma(t)$ , are known from the experimental data. Thus,  $Z(t)$  can be calculated from the second part of Equation 5 as

$$Z(t) = \sigma(t) \left\{ \frac{2n}{n+1} \ln \left[ \frac{2}{\sqrt{3}} D_0 / |\dot{\epsilon}^P(t)| \right] \right\} \frac{1}{2n} \quad (10)$$

for some choice of the constant  $n$ . Experience with the theory shows that changes in  $n$  change the strain rate sensitivity of the prediction by scaling the family of stress-strain rate curves on the abscissa axis but maintain the same shape. Thus, the value of  $n$  and the  $Z$  history can be determined from two stress strain response curves at the higher values of strain rate (to neglect recovery effects) by choosing the value of  $n$  for which the  $Z$  histories from each curve are the same.

The relative rate insensitivity of René 95 at the higher strain rates, as discussed in Reference 1, made the response relatively insensitive to the choice of  $n$  about 3.0. Thus, for convenience in comparing results the functions  $Z(t)$  and  $W_p(t)$  were determined from an experimental stress-strain curve Reference 1 using  $n = 3.2$ . The function  $Z(W_p)$  is shown in Figure 4. Also shown in Figure 4 is Equation 6 using the constants from Reference 12. The correlation with the data is not very good. Alternatively, the model of  $Z(W_p)$

was constructed using both Equations 6 and 9, and the result is also shown in Figure 4. This provides a much better correlation. But, it is interesting to note that  $Z$  vs  $\log W_p$  is plotted, a linear relationship is obtained as shown in Figure 5. This suggests, at least for René 95 that

$$Z(W_p) = \begin{cases} a + b W_p & W_p \leq W_p^0 \\ Z_0' + Z_1' \log W_p & W_p \geq W_p^0 \end{cases} \quad (11)_1$$

$$(11)_2$$

would give a better representation for  $Z(W_p)$ . The linear term,  $(11)_1$ , for small values of plastic work,  $W_p \leq W_p^0$ , is necessary to give the proper asymptotic values as  $W_p$  approaches zero. The values of the constants are

$$Z_0 = 340.0 \text{ KSI} \quad a = 232.0 \text{ KSI}$$

$$Z_1 = 31.12 \text{ KSI} \quad b = 60.68$$

$$W_p^0 = 0.0265 \text{ KSI}$$

The predicted stress-strain response using Equation 11 is shown in Figure 6. A considerable improvement in the shape of the response has been obtained as well as developing a systematic method to obtain above constants. Note, that the predicted creep response was not significantly changed by the use of Equations 11.

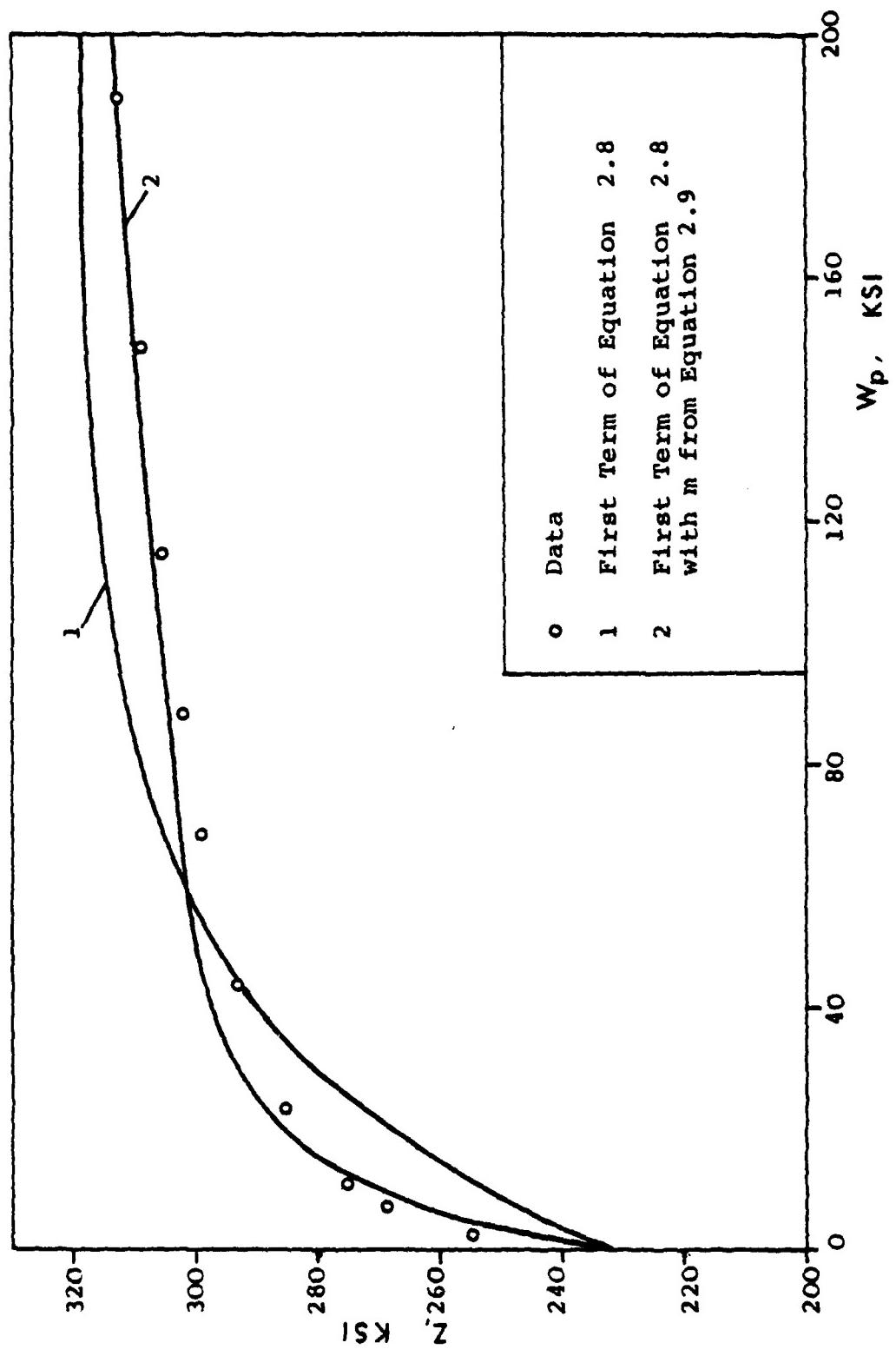


Figure 4. Comparison of the state variable history  $z(w_p)$  with two proposed models.

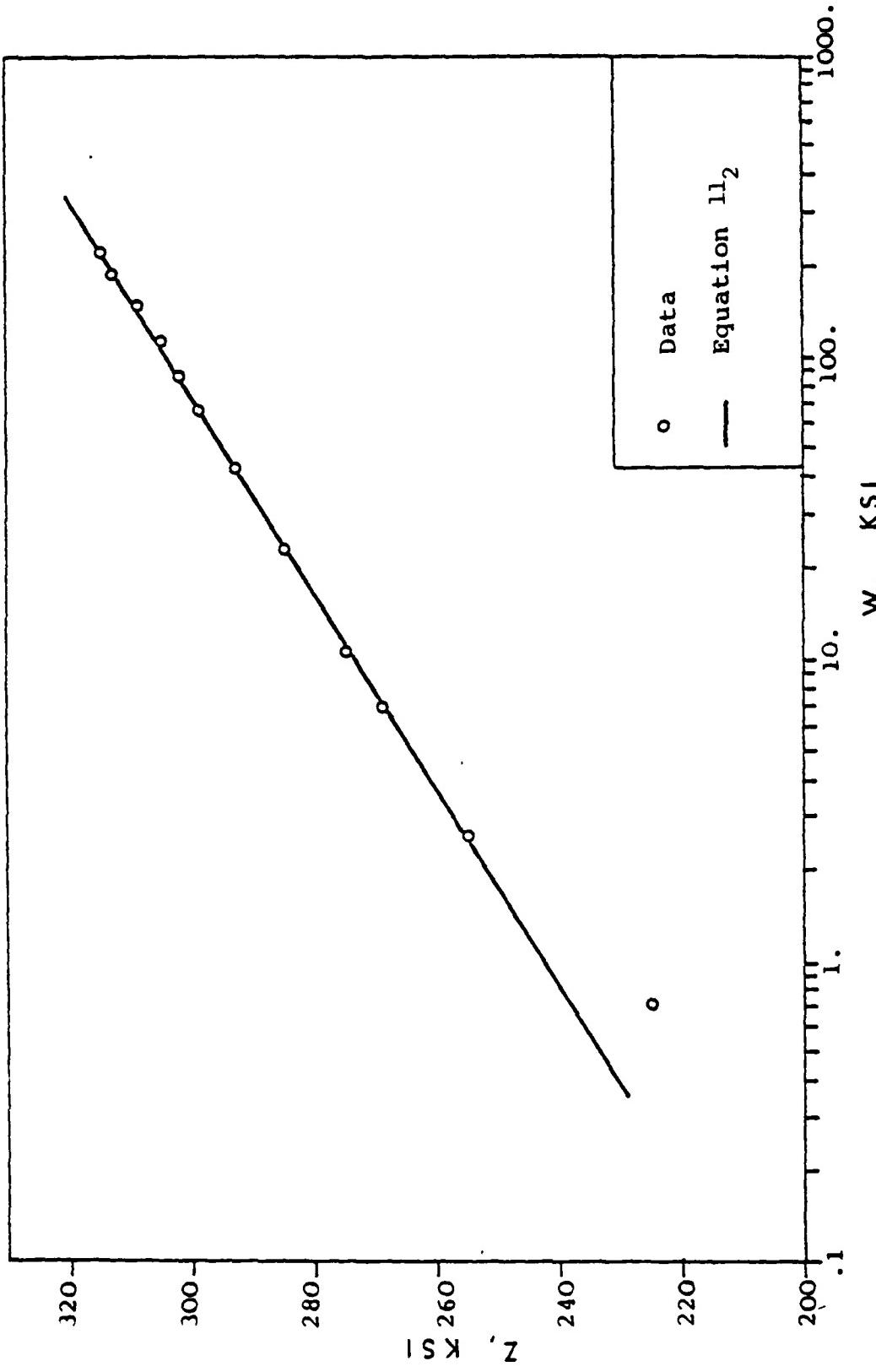


Figure 5. Reformulation of the representation for the state variable history  $z(w_p)$ .

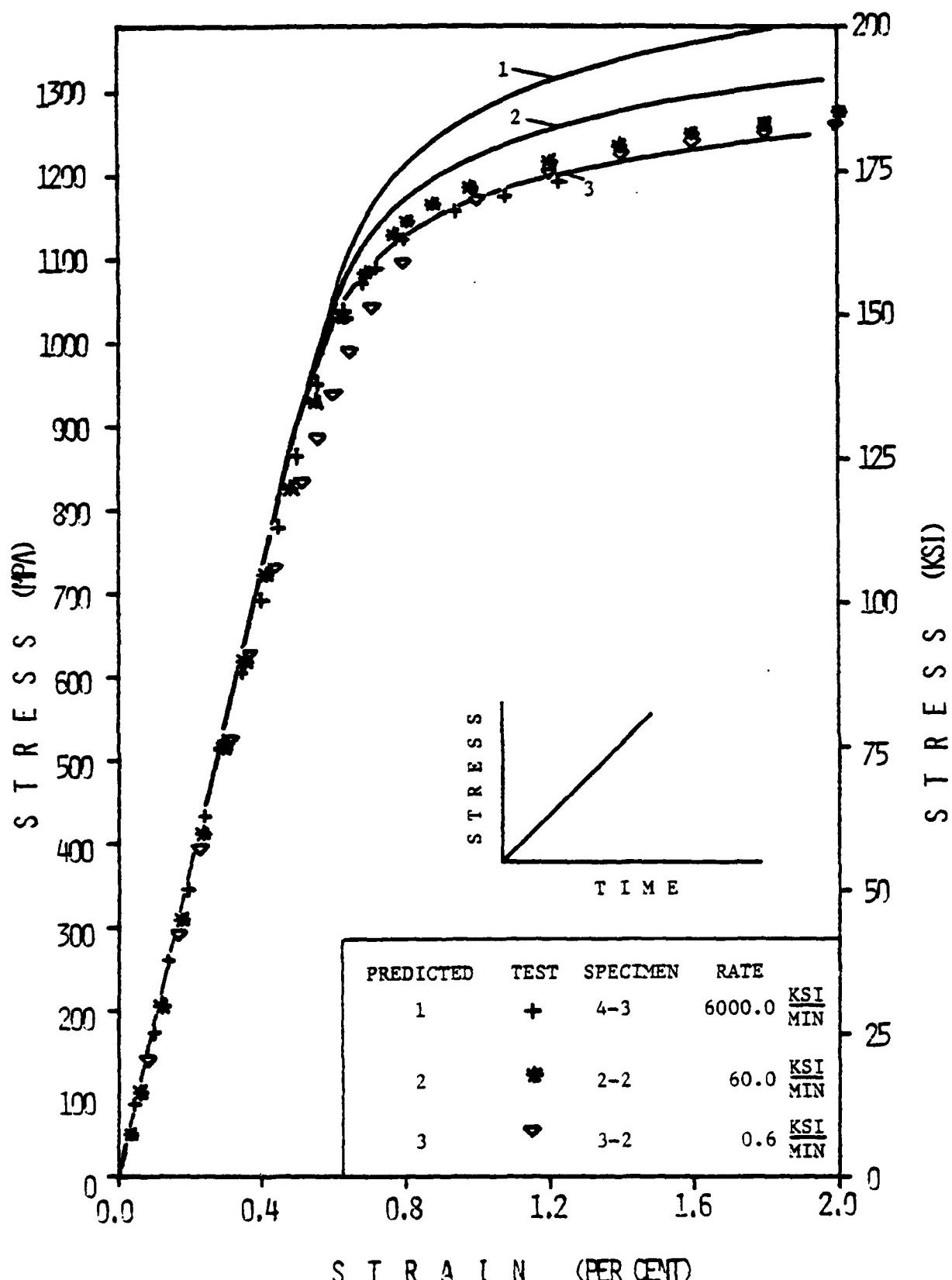


Figure 6. Experimental and predicted stress-strain response using the reformulated state variable representation.

## AN ISOCHRONOUS THEORY

### SECTION V

#### THE CONSTITUTIVE FORMULATION

This approach is based on the Rabotnov nonlinear single integral equation for viscoplasticity, Reference 7. The constitutive equation in a one dimensional case takes the form

$$\phi[\varepsilon(t)] = \sigma(t) + \int_0^t K(t-\tau)\sigma(\tau)d\tau , \quad (12)$$

where  $\phi$  is a nonlinear function of the total strain,  $\varepsilon(t)$ , at the current time,  $t$ , and  $K(t) > 0$  is a measure of the monotonic creep function. The inverse of Equation 12 can be written as, Reference 8,

$$\sigma(t) = \phi[\varepsilon(t)] - \int_0^t R(t-\tau)\phi[\varepsilon(\tau)]d\tau . \quad (13)_1$$

The relaxation function,  $R(t)$ , is the resolvent kernel of  $K(t)$ . Obviously, if  $\phi[\varepsilon(t)]$  is replaced by  $E\varepsilon(t)$  in Equation 12 or 13 the representation reduces to linear viscoelasticity where  $E$  is the elastic modulus. However, to clarify the nonlinearity of the model consider a creep history when  $\sigma(t) = \sigma_0$ , a constant. Equation 12 becomes

$$\frac{\phi[\varepsilon(t)]}{\sigma_0} = 1 + \int_0^t K(t-\tau)d\tau = f(t) \quad (13)_2$$

that is the strain function normalized by the stress,  $\phi[\varepsilon(t)]/\sigma_0$ , becomes a function of time alone. Thus, the theory is applicable only if the isochronous creep curves are similar; that is, if they can be obtained from a master curve by scaling the ordinate at each fixed time. This similarity can be observed in

Figure 7 for René 95 at 650°C(1200°F). This similarity also exists for many other materials as shown in Reference 14.

The function  $\phi[\varepsilon]$  is found from the family of isochronous creep curves or stress-strain curves obtained at various rates in tension and compression. In general,  $\phi[\varepsilon]$  represents a hypothetical state of "instantaneous" loading response which can never be achieved in real experimentation.

Since the constitutive Equation 12 is easy to invert, the relaxation behavior of the material when  $\varepsilon(t) = \varepsilon_0$ , a constant, is governed by Equation 13 which takes the form

$$\frac{\sigma(t)}{\phi(\varepsilon_0)} = 1 - \int_0^t R(t-\tau)d\tau = f_1(t); \quad (14)$$

that is, the stress normalized by the strain function becomes a function of time alone. If the creep function  $K(t)$  is chosen in the form of some analytical expression, the relaxation function  $R(t)$  can be found by means of the integral equation theory.

A successful representation for the kernel function  $K(t)$  for René 95 has been found in the form of power law

$$K(t-\tau) = \frac{A}{(t-\tau)^\alpha}, \quad (15)$$

where  $A$  and  $\alpha$  are the material constants. The constant  $\alpha$  is restricted to the  $0 < \alpha < 1$ , which gives a weak (integrable) singularity at  $\tau = t$  that is easily overcome numerically. The resolvent function  $R(t)$  in this case takes the form of fractional-exponential function

$$E_\alpha(-\beta, t-\tau) = \sum_{n=0}^{\infty} \frac{(-\beta)^n (t-\tau)^{(\alpha+1)n+\alpha}}{\Gamma[(\alpha+1)(n+1)]}, \quad (16)$$

where  $\beta = \alpha \Gamma (\alpha + 1)$ . The properties of the function  $E_{\alpha}(-\beta, t)$  are given in Reference 18.

Nonlinear hereditary equations of this type describe active deformation processes when the load is a nondecreasing function of time. Accordingly, a condition of the applicability of the above equation is  $\dot{\sigma}(t) > 0$ . However, the model in question makes it possible to obtain a simple constitutive equation for unloading in tension. If the elastic unloading is assumed, this equation takes the form

$$E[\varepsilon^* - \varepsilon(t)] + \phi^* = \sigma(t) + \int_0^t K(t - \tau)\sigma(\tau)d\tau, \quad \dot{\sigma}(t) < 0, \quad (18)$$

where  $E$  is the instantaneous modulus of elasticity,  $\varepsilon^* = \varepsilon(t^*)$  is the value of maximum strain achieved in the loading process at time  $t^*$ , and  $\phi^*$  is the value of  $\phi[\varepsilon(t^*)]$ . Combining Equations 12 and 13, a single general tensile load-unload equation can be written as, Reference 8,

$$F[\varepsilon(t), \varepsilon^*] = \sigma(t) + \int_0^t K(t - \tau)\sigma(\tau)d\tau, \quad (19)$$

where the instantaneous load-unload master curve is

$$F[\varepsilon(t), \varepsilon^*] = \begin{cases} \phi[\varepsilon(t)] & \text{when } \dot{\sigma}(t) \geq 0 \\ E[\varepsilon^* - \varepsilon(t)] + \phi^* & \text{when } \dot{\sigma}(t) < 0. \end{cases}$$

Equation 19 is easily extended to compressive histories as shown in the next section. In the above equation function  $K(t)$  takes into account the entire history of stress  $\sigma(t)$  from the moment of application of the load. The process

described as active deformation (see Figure 8) corresponds to loading upwards along the  $\sigma = \phi(\varepsilon)$  curve as far as the strain  $\varepsilon^*$  and unloading downwards along the straight line  $\sigma = E(\varepsilon^* - \varepsilon(t)) + \phi^*$ . In this case it is possible to take into account the strain recovery. Equation 19 thus makes it possible to describe the deformation response to different histories, including cyclic loading and the strain recovery following removal of the load.

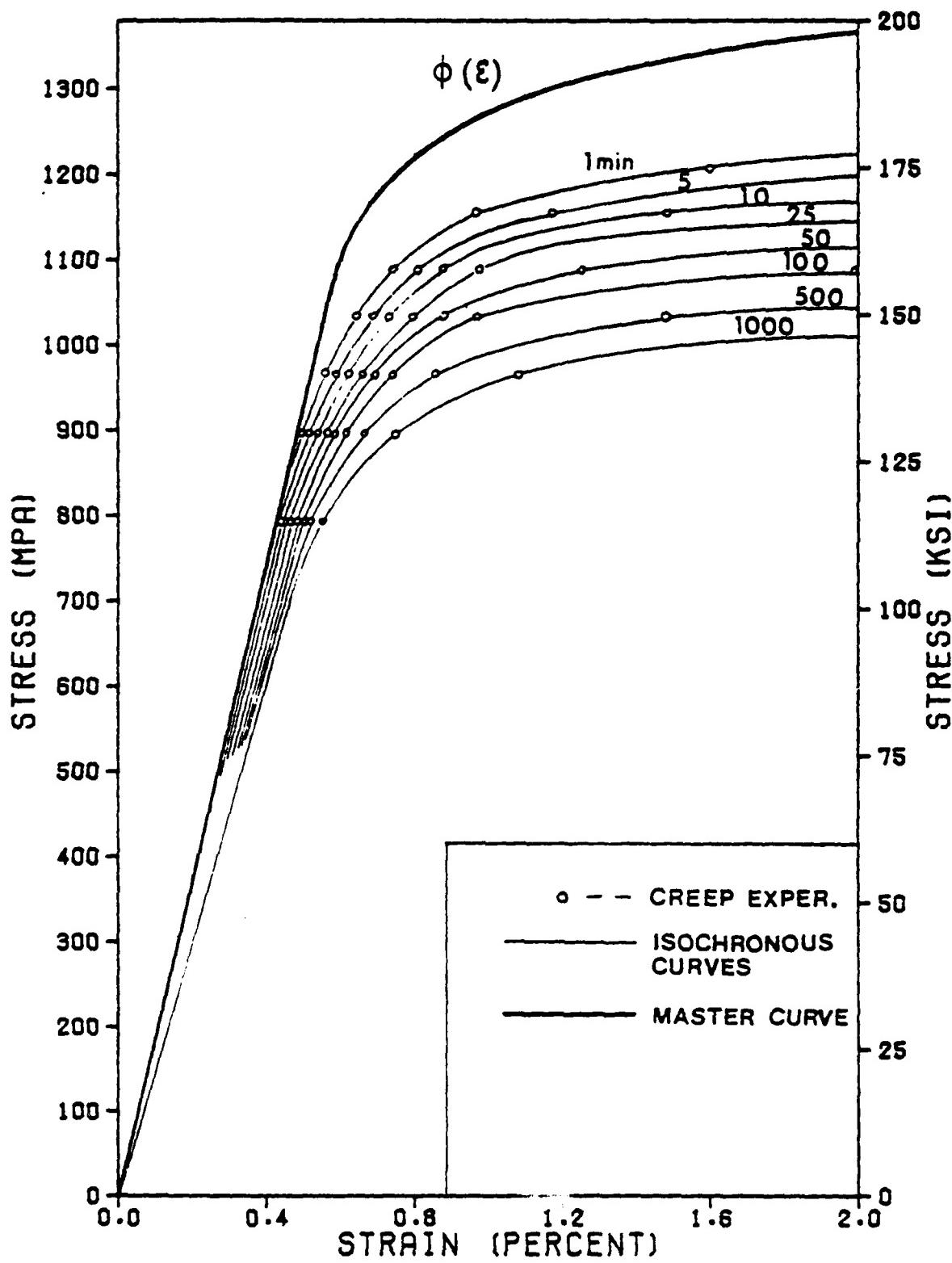


Figure 7. Isochronous tensile creep curves for Rene' 95 at 650°C (1200°F) and constructed master loading curve

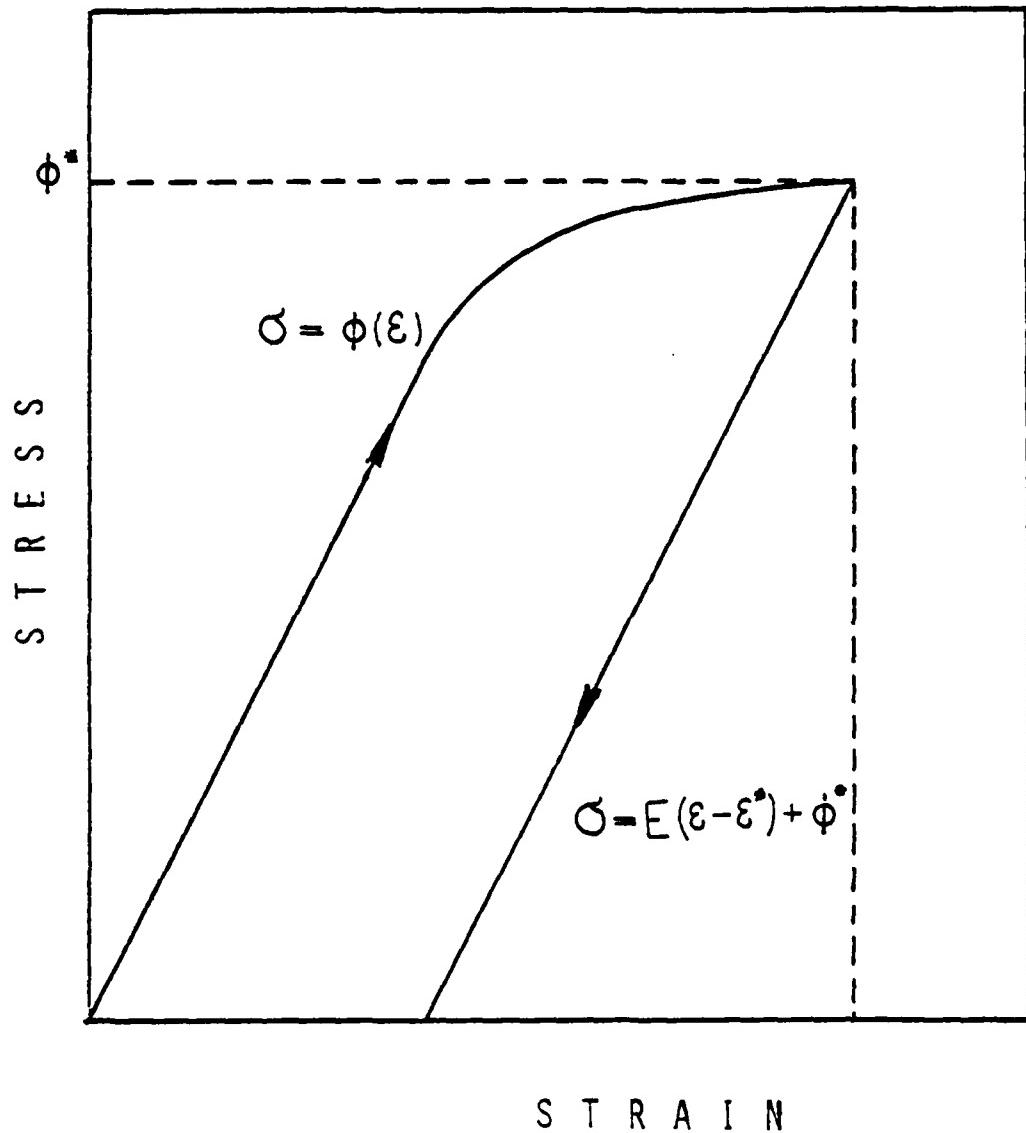


Figure 8. Definition of the master loading and unloading curves for the isochronous theory

## SECTION VI

## APPLICATION TO RENE 95

To apply the model to a material it is necessary to evaluate the constants and A in the kernal function K(t); and, to develop a representation for the strain function  $\phi[\epsilon]$ .

The procedure for evaluating  $\alpha$  and A is accomplished from the data in the isochronous creep curves. To begin, let  $\sigma(t) = \sigma_0$ , a constant, and substitute Equation 16 into 12 to get

$$\phi[\epsilon(t)] = \sigma_0 [1 + A_1 t^{1-\alpha}] \quad (20)$$

where  $A_1 = A/(1-\alpha)$ . Define some isochrone  $t=t_0$  of the family as the basis and let the strain  $\epsilon_0$  correspond to  $\sigma_0$  at  $t_0$ . Next construct the ratio

$$1 = \frac{\phi(\epsilon_0)}{\phi(\epsilon)} = \frac{\sigma_0 [1 + A_1 t_0^{1-\alpha}]}{\sigma_0 [1 + A_1 t^{1-\alpha}]} \quad (21)$$

where  $\phi(\epsilon_0)$  is the fixed strain function defined on another isochrone at the creep stress  $\sigma$ . Rewriting Equation 21 gives

$$\frac{\sigma_0}{\sigma} = b(1 + A_1 t_0^{1-\alpha}), \quad (22)$$

where

$$b = \frac{1}{1 + A_1 t_0^{1-\alpha}} \quad (23)$$

Equation 22 is linear in the coordinates  $\sigma_0 / \sigma$  and  $t^{1-\alpha}$  for any strain  $\varepsilon_0$ . Therefore, the family in coordinates  $\sigma_0 / \sigma$  and  $t^{1-\alpha}$ , corresponding to  $\alpha$  for each isochrone  $t_0$ , can be brought into a straight line for the proper choice of  $\alpha$ . In this case, the point of intersection of the straight line and the ordinate axis gives the value of  $b$ , and the slope is used to determine the value of  $A_1$ . This procedure is illustrated in Figure 9 for  $\varepsilon_0 = 2.0\%$  and the resulting values are given in Table I. To obtain the best average values for  $b$  and  $\alpha$ , this process should be repeated for other choices of  $\varepsilon_0$  using the same basis  $t_0$ ; and, also for other choices of the basis  $t_0$ . This was done for René 95 and the average values are

$$\alpha = 0.83 \quad \text{and} \quad A = 0.019.$$

Observe that points of the master curve  $\phi[\varepsilon(0)]$  at time  $t = 0$  can be determined from Equation 22 for each base curve  $\sigma_0$  and  $t_0$ . Thus the master curve is derived from the extrapolated point  $b$ . The master curve should therefore be considered as a hypothetical instantaneous response function.

To establish a representation of the strain function  $\phi[\varepsilon(t)]$ , it is advantageous to observe that the right hand side of Equation 12 represents a pseudo stress  $\hat{\sigma}(t)$  that depends on the actual stress history  $\sigma(\tau)$  for  $\tau \in [0, t]$  and contains all the hereditary information. Thus Equation 12 can be written as

$$\phi[\varepsilon(t)] = \hat{\sigma}(t) \quad (24)$$

where

$$\delta(t) = \sigma(t) + A \int_0^t \frac{\sigma(\tau) d\tau}{(t - \tau)^{1-\alpha}}. \quad (25)$$

Further, since Equation 12 is assumed to uniquely describe the strain function for any stress history,  $\phi[\varepsilon(t)]$  possesses a unique inverse

$$\varepsilon(t) = \phi^{-1}[\hat{\sigma}(t)] \quad (26)$$

Thus, using Equations 25 and 26 the history dependence can be represented by a pseudo stress-strain equation.

A specific representation for René 95 can be established by using the Ramber-Osgood equation, Reference 15, for  $\phi^{-1}[\hat{\sigma}(t)]$ . Let

$$\varepsilon(t) = \theta^{-1}[\hat{\sigma}(t)] = \frac{\hat{\sigma}(t)}{E} + \left[ \frac{\hat{\sigma}(t)}{M} \right]^{\frac{1}{m}} \quad (27)$$

where E is the elastic modulus of the material, M and m are constants that must be determined from the master curve  $\sigma(0) = \phi[\varepsilon(0)]$ . The values  $M = 1351 \text{ MPa}$  (196 KSI) and  $m = 0.05$  were found for René 95 at  $650^\circ\text{C}$ . ( $1200^\circ\text{F}$ ).

The stress strain curves at different stress rates and the creep curves calculated using Equations 27 and 25 are shown in Figures 10, 11, and 12. The model exhibits a rate sensitivity which is determined by a spacing of three solid lines in Figure 10 representing change of strain rate of  $10^4$ . René 95 does not exhibit the same magnitude of rate sensitivity in this range. However, good approximation of the creep curves makes it reasonable to use this simple approach for a description of the material behavior.

TABLE I. COEFFICIENTS FOR THE ISOCHRONOUS CREEP THEORY

$t_0$ min	5% MPa (KSI)	$A_1$	b
1	1224.6 (177.6)	0.111	0.898
50	1124.5 (163.1)	0.107	0.827
1000	1043.9 (151.4)	0.112	0.735

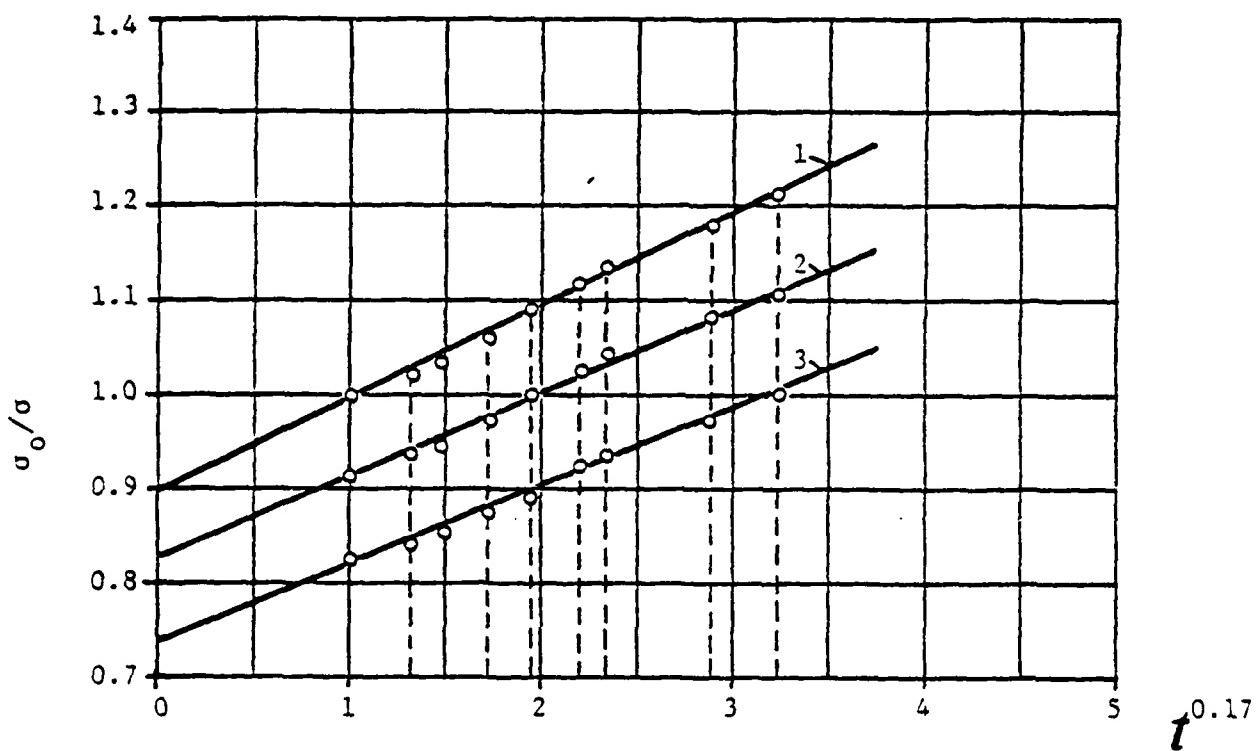


Figure 9. Determination of the values of  $\alpha$  and  $A_1$  for  $\epsilon_0 = 2.0\%$  and times:  
 (1)  $t_0 = 1.0$  min, (2)  $t_0 = 50$  min, (3)  $t_0 = 1000$  min

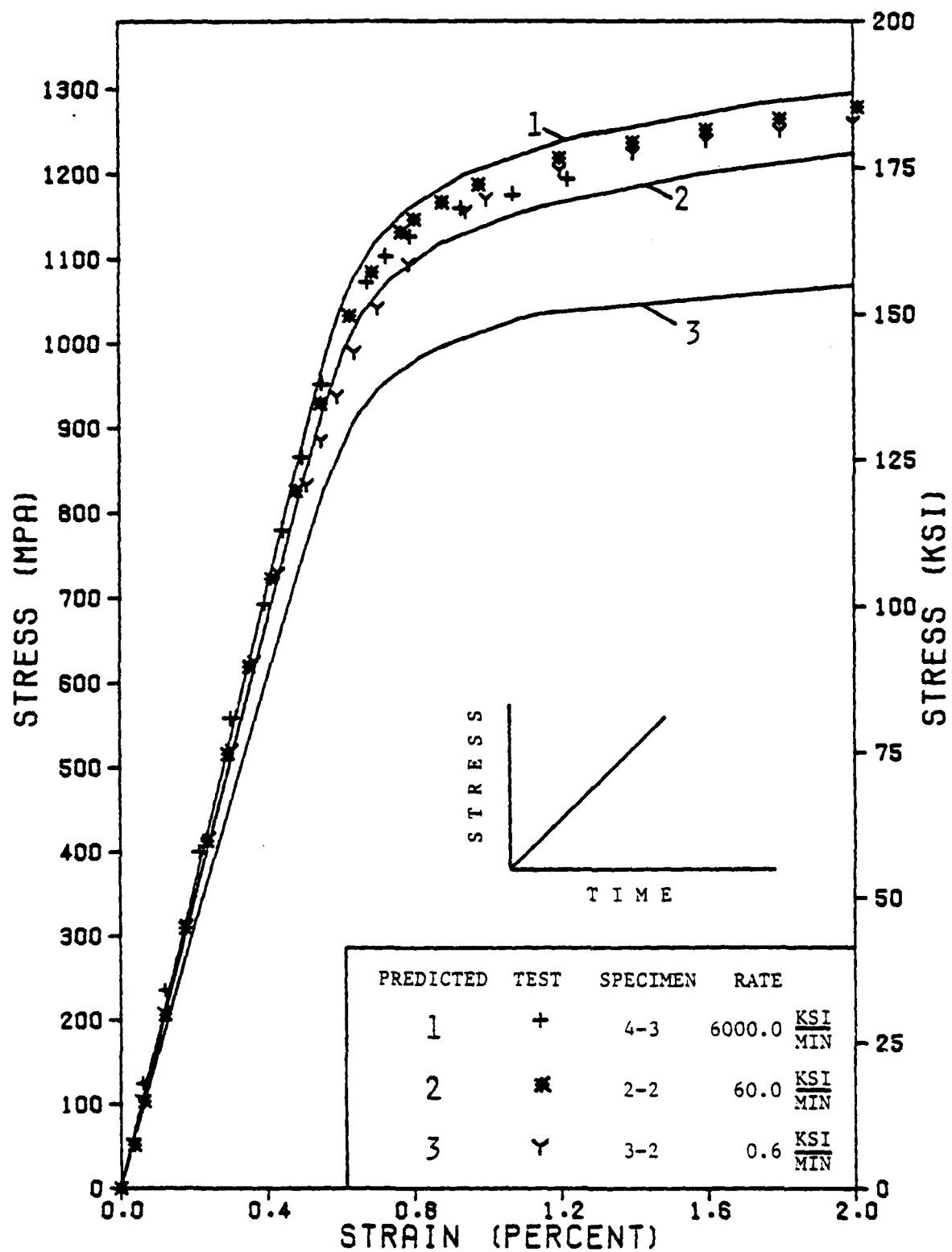


Figure 10. Experimental and predicted stress-strain response using the isochronous theory

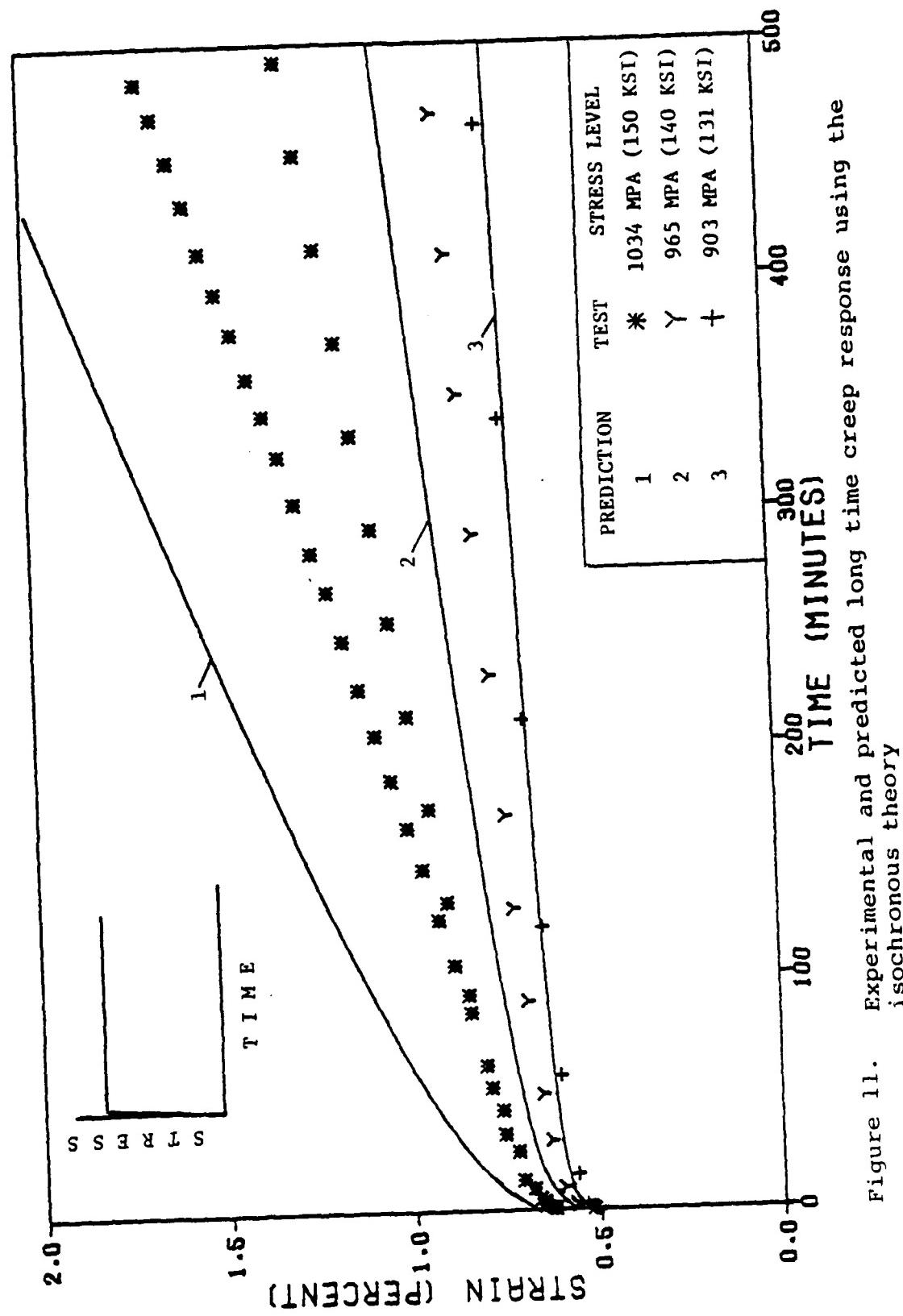


Figure 11. Experimental and predicted long time creep response using the isochronous theory

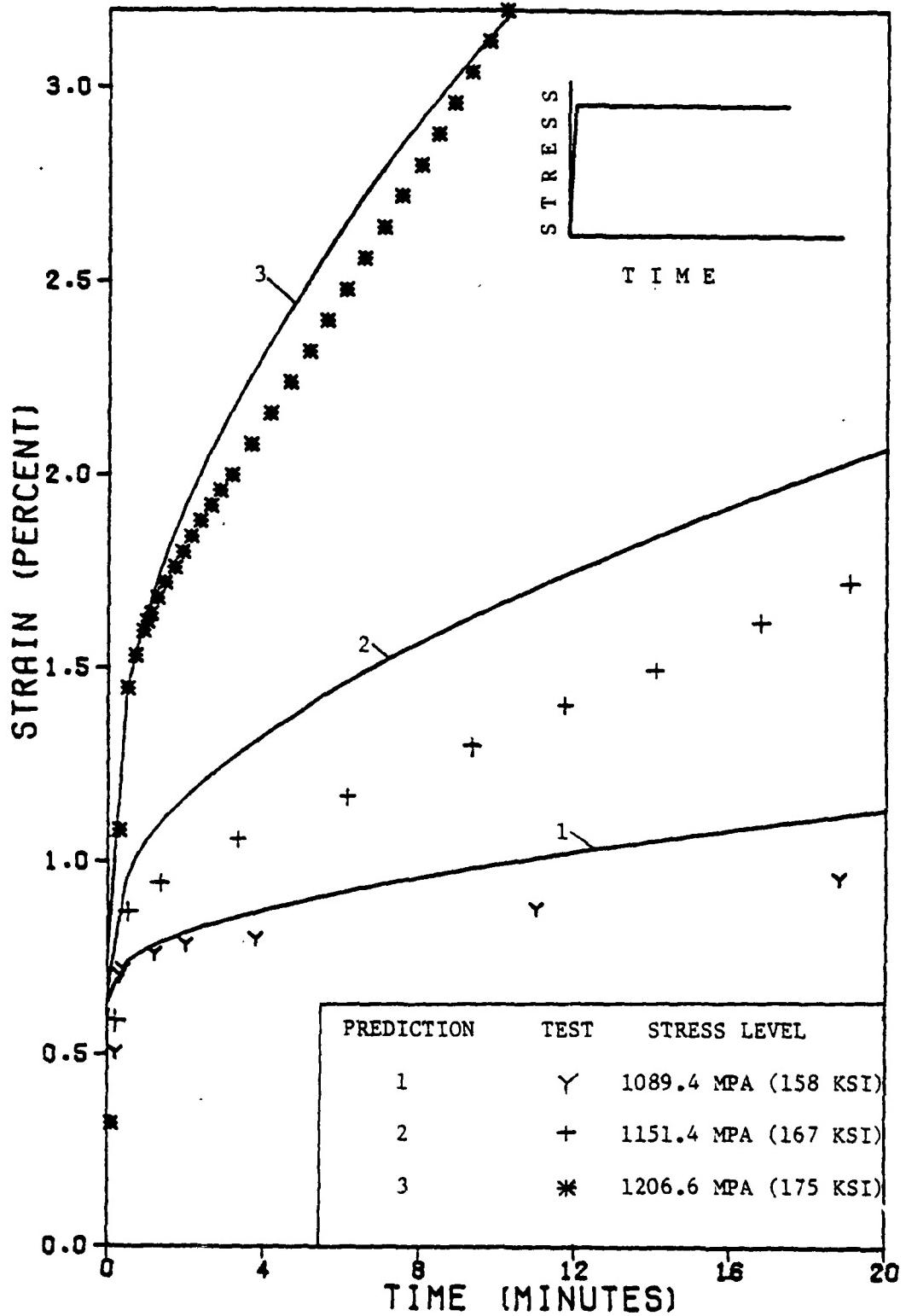


Figure 12. Experimental and predicted short time creep response using the isochronous theory

## A NONLINEAR CREEP FORMULATION

### SECTION VII

#### PROPERTIES OF THE CONSTITUTIVE THEORY

Most of the classical methods of predicting creep are based on integral type constitutive equations constructed from or equivalent to nonlinear "superposition" type arguments. As such, they fail to accurately predict strain recovery in metals at elevated temperature. This and other considerations suggest the following approach for developing a successful viscoplasticity constitutive law:

- (1) The formulation should, at least in part, be developed directly from some important experimentally determined function. This is to avoid material functions with no physical meaning.
- (2) Since elevated temperature material response is usually rate (time) dependent, a constitutive formulation similar to viscoelasticity is appropriate, however; the approach must be modified to predict the correct anelastic recovery properties for metals.
- (3) Establish and experimentally verify a one dimensional constitutive theory. Then, if the material is isotropic, homogeneous, and isochoric, a three dimensional model can be theoretically developed with a minimum of one scalar material function. (It is unlikely that such a model could predict all of the material memory effects that would be observed in multiaxial testing. However, once developed, such a model would help identify which material memory effects need additional representation.)

To begin, let us select an equation that will adequately predict the constant stress creep response of metals. The results of the experiments in Reference 1 can be collectively modeled by the Marin-Pao equation, Reference 17.

$$K[\sigma, t] = A(\sigma)[1 - \exp(-\beta(\sigma)t)] + \dot{\epsilon}_{\min}(\sigma)t, \quad (28)$$

where  $K[\sigma, t]$  is the uniaxial creep function. The minimum creep rate is denoted by  $\dot{\epsilon}_{\min}$ , and all three coefficients are functions of the creep stress  $\sigma$ .

Next, assume that the response characteristics described by the constant load creep test, Equation 28 or equivalent, must be contained in any general constitutive representation for a time varying stress history. Further, thermodynamic coordinates  $q$ , can be introduced into the formulation to account for the history and memory effects. Furthermore, it can be assumed that the time argument should be replaced by a "material clock",  $\zeta$ . In Reference 6, a complete development was given which included the  $q$  and  $\zeta$ . However, it was found that it was not necessary to include these terms in order to represent the material response examined herein. Therefore, the simpler representation is outlined below.

The representation given in Equation 28 or equivalent can be extended to include transient stress histories,  $\sigma(\tau)$  for  $\tau \in (-\infty, \infty)$ . This extension rests on the assumption:

The amount of creep that occurs in some infinitesimal increment of time  $[\tau, \tau + \Delta \tau]$  depends only on the mean value of stress, temperature, and a measure of the material state present during that increment of time.

This assumption allows a representation for transient stress histories to be established by partitioning  $\sigma(t)$  for  $t \in [t_0, t]$  into  $N$  subintervals, evaluating the response in each interval, and integrating to obtain the total response. Let  $t_i$  ( $i = 1, 2, \dots, N$ ) be the time at the beginning of the  $i^{\text{th}}$  time interval and let  $\sigma_i$  be the average values of stress during the  $i^{\text{th}}$  time interval.

The increment of strain at any time  $\tau$  due to a stress pulse  $\sigma_i$  during the time interval  $[\tau_i, \tau_i + \Delta\tau_i]$  is assumed to be given by the Equation 28 applied at time  $\tau_i$  and subtracted at time  $\tau^*$ , where

$$\tau^* = \tau_i + \alpha \Delta\tau_i \quad (29)$$

for  $0 \leq \alpha \leq 1$ . The variable  $\alpha$  is a material function that allows for varying amounts of anelastic recovery to be included in the model. In general,  $\alpha$  will be a path and time dependent state variable. Proceeding to construct an integral using a method similar to linear viscoelasticity, Reference 6, gives a representation for the inelastic strain as

$$\varepsilon^I(t) = \int_0^t \alpha \frac{\partial}{\partial t} K[\sigma(\tau), t - \tau] + (1 - \alpha) \left. \frac{\partial}{\partial \xi} K[\sigma(\tau), \xi] \right|_{\xi=0} d\tau. \quad (7.3) \quad (30)$$

The total strain,  $\varepsilon(t)$ , is then given by

$$\varepsilon(t) = \frac{\sigma(t)}{E} + \varepsilon^I(t) \quad (31)$$

where  $E$  is the elastic modulus.

Let us consider the effect of the material parameter  $\alpha$  on the range of values  $0 \leq \alpha \leq 1$ . If  $\alpha = 1$ , Equations 30 and 31 corresponds to the viscoelasticity theory of Stouffer, Reference 16, where all primary creep is anelastic and therefore recoverable. Also, for a constant stress history, Equation 30 yields the creep Equation 28. If  $\alpha = 0$ , then Equation 31 becomes

$$\varepsilon(t) = \int_0^t \left. \frac{\partial \varepsilon [\sigma(\tau), \xi]}{\partial \xi} \right|_{\xi=0} d\tau. \quad (32)$$

In this case, the model predicts that the creep is permanent for all time (i.e., non-recoverable) as in plasticity. The result for  $\alpha = 0$  also corresponds to the response during high stress rate loading. Thus  $\alpha$  is a viscoplasticity parameter that controls the relative contribution of the viscoelastic and plastic components to the total inelastic strain.

As shown through the previous discussion, Equation 30 is sufficiently general to model the spectrum of deformation response features characteristic of materials at elevated temperatures. However, during an arbitrary stress history, a method is needed which will translate the current stress condition into a value of  $\alpha$ . For example, if the stress is constant, then  $\alpha$  should be unity in order to predict the creep curve response. Conversely, if the material is experiencing a rapid change in stress, then the response should correspond to a plastic deformation and  $\alpha$  should approach zero. However, if anelastic recovery occurs in the material during a rapid stress transient, then a method of accounting for this type of response is needed and  $\alpha$  cannot be exactly zero. These considerations are used to develop a representation for  $\alpha$ . It is shown in Reference 6, that  $\alpha$  is a function of the history of the stress rate and that must also satisfy certain continuity requirements.

SECTION VIII  
APPLICATION TO RENE 95

To use the above system of equations for a real material it is necessary to develop representations for the functions  $A(\sigma)$ ,  $B(\sigma)$  and  $\dot{\epsilon}_m(\sigma)$  in Equation 28 along with the parameter  $\alpha$ . At the outset of the program it was decided to use the simplest form for  $\alpha$ , namely

$$\alpha[\dot{\sigma}(t)] = \begin{cases} 1 & \text{for } \dot{\sigma} \approx 0 \\ 0 & \text{for } \dot{\sigma} \neq 0. \end{cases} \quad (33)$$

This provides the opportunity to investigate the interaction between the first and second terms in Equation 30. In general, to predict creep,  $\alpha = 0$  during the initial load to the creep stress and  $\alpha = 1$  during the constant stress portion of the creep history. Conversely,  $\alpha = 0$  for the duration of a constant stress rate test in tension or compression. This approximation is reasonable for René 95 for stress rate above 7 MPa/min (1.0 KSI) since there is very little effect of the stress rate on the response. (See Figure 6 of Reference 1). However, if the rate processes are in the creep domain, as shown in Figure 6 of Reference 1, then Equation 33 would not be expected to predict the correct response. This deficiency could easily be corrected with a set of experiments at very low constant stress to determine  $\alpha[\dot{\sigma}]$ .

The minimum creep rate function,  $\dot{\epsilon}_m(\sigma)$  and the magnitude of the primary creep  $A(\sigma)$ , can be directly determined from creep response data as given in Table 2. A representation of this data is given by

$$A(\sigma) = \frac{1}{100} \frac{\sigma}{|\sigma|} \exp[\exp(a_1 + a_2\sigma)], \quad (34)$$

$$\dot{\epsilon}_m(\sigma) = \frac{\sigma}{|\sigma|} \exp(c_1 + c_2\sigma). \quad (35)$$

The time parameter was found to have the form

$$\beta(\sigma) = b_1 \left( \frac{\sigma}{100} \right)^{b_2} \quad (36)$$

from the onset of secondary creep. However, the constant,  $b$ , was picked to give the best prediction of the response in tension and compression. Also, it is expected that the coefficients  $a_1$ ,  $a_2$ ,  $b_1$ ,  $b_2$ ,  $c_1$  and  $c_2$  would be different in tension and compression, however; sufficient data is not available to totally determine all the compression coefficients. Thus, the same values for tension and compression are used in both cases. The values used for René 95 are given in Table 3.

The reproduction of the creep and tensile data using Equation 30 is shown in Figures, 13, 14, and 15. Since the model does not predict strain rate effects for the current choice of  $\alpha$ , only one curve is shown in Figure 13 with the nominal experimental response. The accuracy is certainly within the consistency of the experimental data. In all cases, the shape and magnitudes of the predicted curves match the response quite well.

Another fundamental property of the model that should be included in this section is the response on unloading. In general, Equation 30 is a creep formulation that will predict a positive inelastic strain rate for all positive values of stress. This implies that the model cannot predict "elastic" unloading. Thus, Equation 30 is modified such that

$$\dot{\epsilon}^I(\tau) = 0 \quad \text{whenever} \quad (37)$$

$$\frac{\sigma(\tau)}{|\sigma(\tau)|} \dot{\sigma}(\tau) < e$$

where  $e$  is a small positive parameter.

A consequence of Equation 37 is that recovery cannot be predicted by the model if  $\alpha = 0$ . Thus,  $e = 69 \text{ MPa/min}$  ( $10 \text{ KSI/min}$ ) was used for René 95.

TABLE 2. CREEP PROPERTIES OF RENE 95 AT 650°C(1200°F)

Creep Stress MPa (KSI)	Spec Number	Min Creep Rate %/min	Inelastic value of A( $\sigma$ ) %
877.0 (127.2)	2 - 7	$3.21 \times 10^{-4}$	0.111
903.5 (131.0)	3 - 5	$2.76 \times 10^{-4}$	0.120
965.3 (140.0)	2 - 8	$4.4 \times 10^{-4}$	0.133
1034.3 (150.0)	3 - 8	$1.15 \times 10^{-3}$	0.166
1034.3 (150.0)	1 - 5	$2.0 \times 10^{-3}$	0.094
1089.4 (158.0)	1 - 8	$9.09 \times 10^{-3}$	0.163
1156.3 (167.7)	2 - 5	$4.00 \times 10^{-2}$	0.285
1206.6 (175.0)	1 - 7	$1.63 \times 10^{-1}$	0.804

TABLE 3. COEFFICIENTS FOR EQUATIONS 34, 35 AND 36  
FOR RENE 95 AT 650°C(1200°F).

Coefficient	Tension and Compression
$\sigma_1$	- 0.9370
$\sigma_2$ , MPa <sup>-1</sup> (KSI <sup>-1</sup> )	$0.187 \times 10^{-2}$ ( $1.295 \times 10^{-2}$ )
$b_1$ , min <sup>-1</sup>	0.200
$b_2$	15.0
$c_1$	-30.67
$c_2$ , MPa <sup>-1</sup> (KSI <sup>-1</sup> )	0.244 (0.1684)

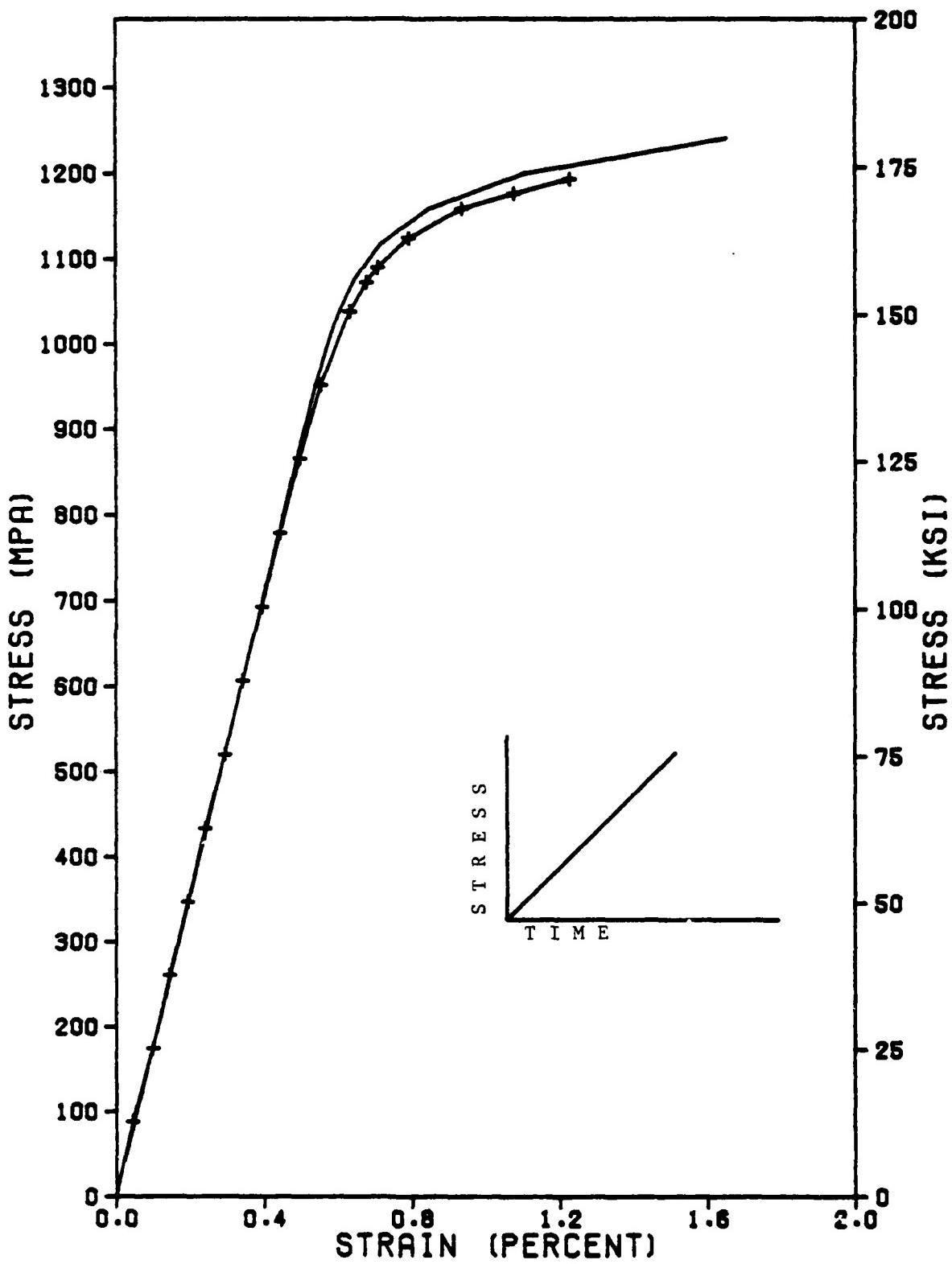


Figure 13. Experimental and predicted stress-strain response using the non-linear creep theory.

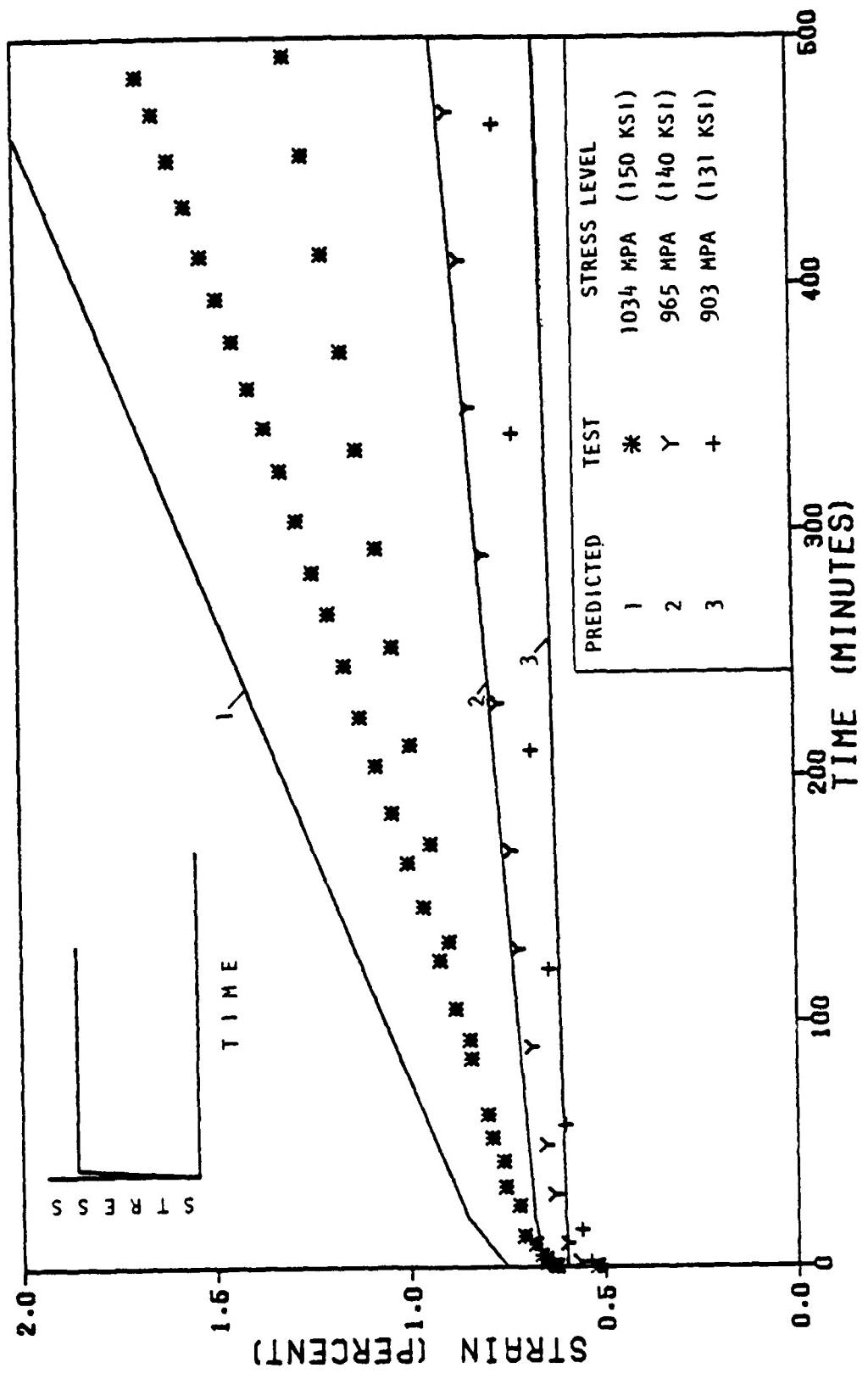


Figure 14. Experimental and predicted long time creep response using the non-linear creep theory

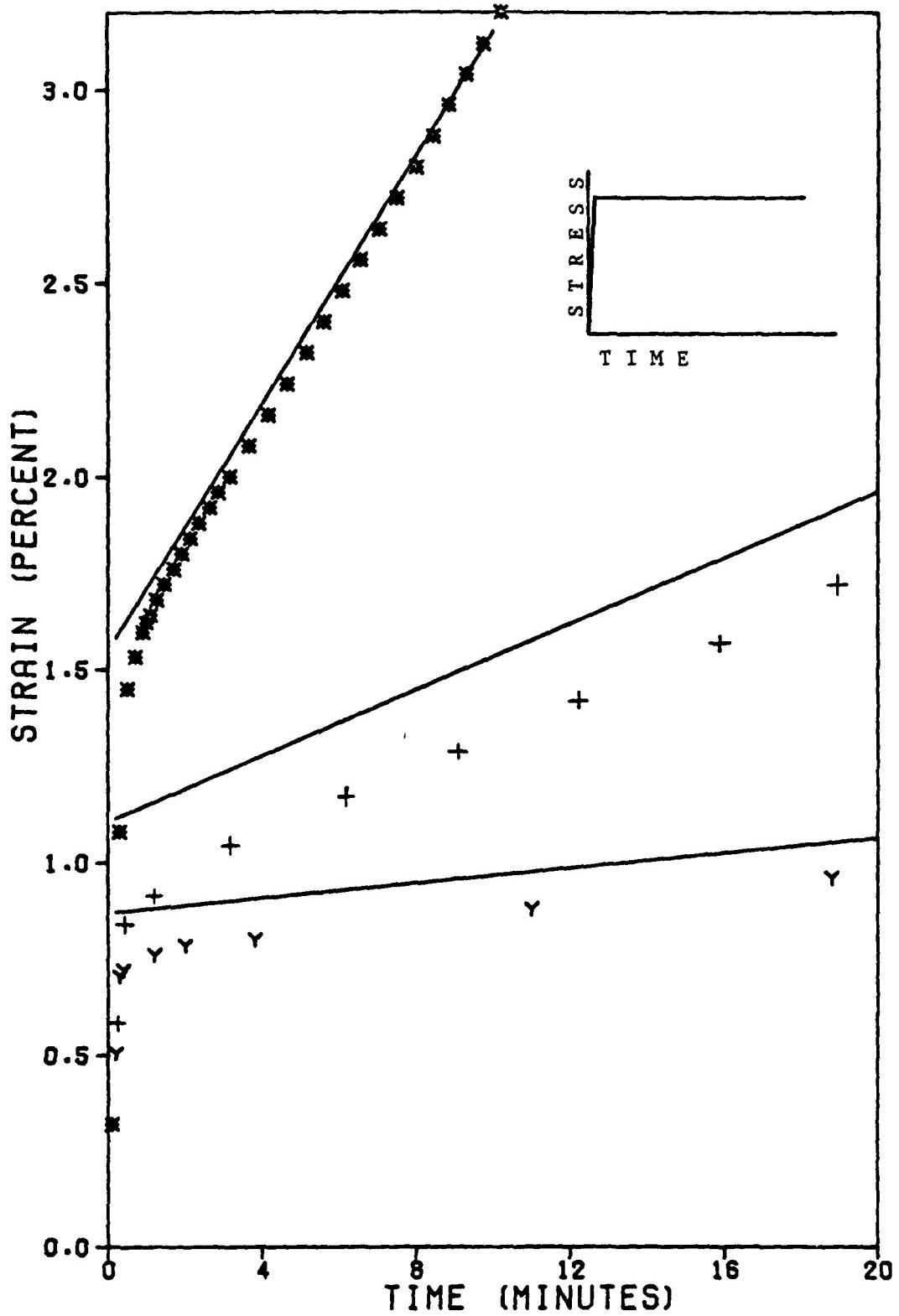


Figure 15. Experimental and predicted short time creep response using the non-linear creep theory.

PREDICTIONS AND RESULTS  
SECTION IX  
THE PREDICTIVE CAPACITY OF THE MODELS

In the three previous parts of this report, the material parameters for the models were determined from the tensile and creep response of the material. In general, it was found that the models could reproduce the response characteristics relatively well, and the accuracy was within the repeatability of the experimental data. The predictive capability of the models depends upon their ability to reproduce the strain response to stress histories distinctly different from the previous set of experiments. Thus, the models were used to predict three different hysteresis loops and stress relaxation.

Let us consider first a simple hysteresis loop under stress control. The stress history is 0, +1151, -1151, 0 MPa (0, +167, -167, 0 KSI) at a rate corresponding to 10 CPM. The predicted results are shown in Figures 16, 17, and 18. It can be seen that the Bodner-Partom model, Figure 16, overpredicts the strain in compression. This results from assuming the response in tension and compression are equal. However, on this particular test, the Lafren-Stouffer model underpredicts the compression. This most likely reflects the difference between response in Figure 17 and the nominal compressive response of the material. The Rabotnov-Papernik prediction appears to be best for this particular test.

Consider next, the response to a similar history except that the material is initially loaded in compression rather than tension; i.e., 0, -1151, +1151, 0 MPa (0, -167, +167, 0 KSI). The constants in all three models were adjusted to predict the same response in tension and compression due to the lack of compression data. Thus, the predicted response can be obtained by rotating Figures 16,

17, and 18 about the origin 180 degrees. However, the relationship between the observed response in tension and compression does not follow this simple rule. Thus, the compressive response must be included in the models to accurately predict the hysteresis response of René 95, especially for several cycles.

Another evaluation was made by comparing the predicted response to the unbalanced hysteresis loop shown in Figures 19, 20, and 21. In this example, the stress history is 0, 1151, -600, 0 MPa (0, 167, -87, 0 KSI), also at a rate equivalent to 10 cpm. The prediction of all three models is approximately equivalent and matches the experimental data relatively well.

As a final example, consider the capability of the models to predict stress relaxation to a 1.0% step strain history. As shown in Figure 22, the Bodner-Partom and Rabotnov-Papernik models are relatively accurate at long times. However, the initial rate of stress relaxation is not predicted very well by either constitutive theory. To avoid inverting Equation 30, the observed stress history from the stress relaxation experiment was used to calculate the corresponding strain history. These results are given in Table 4. It can be seen that the predicted strain history is essentially constant over the entire domain of the experimental data even though the average strain is not 1.0%. However, from the stress-strain curves, Figures 7 and 9 of Reference 1, it can be seen that the error is within the response band of the material.

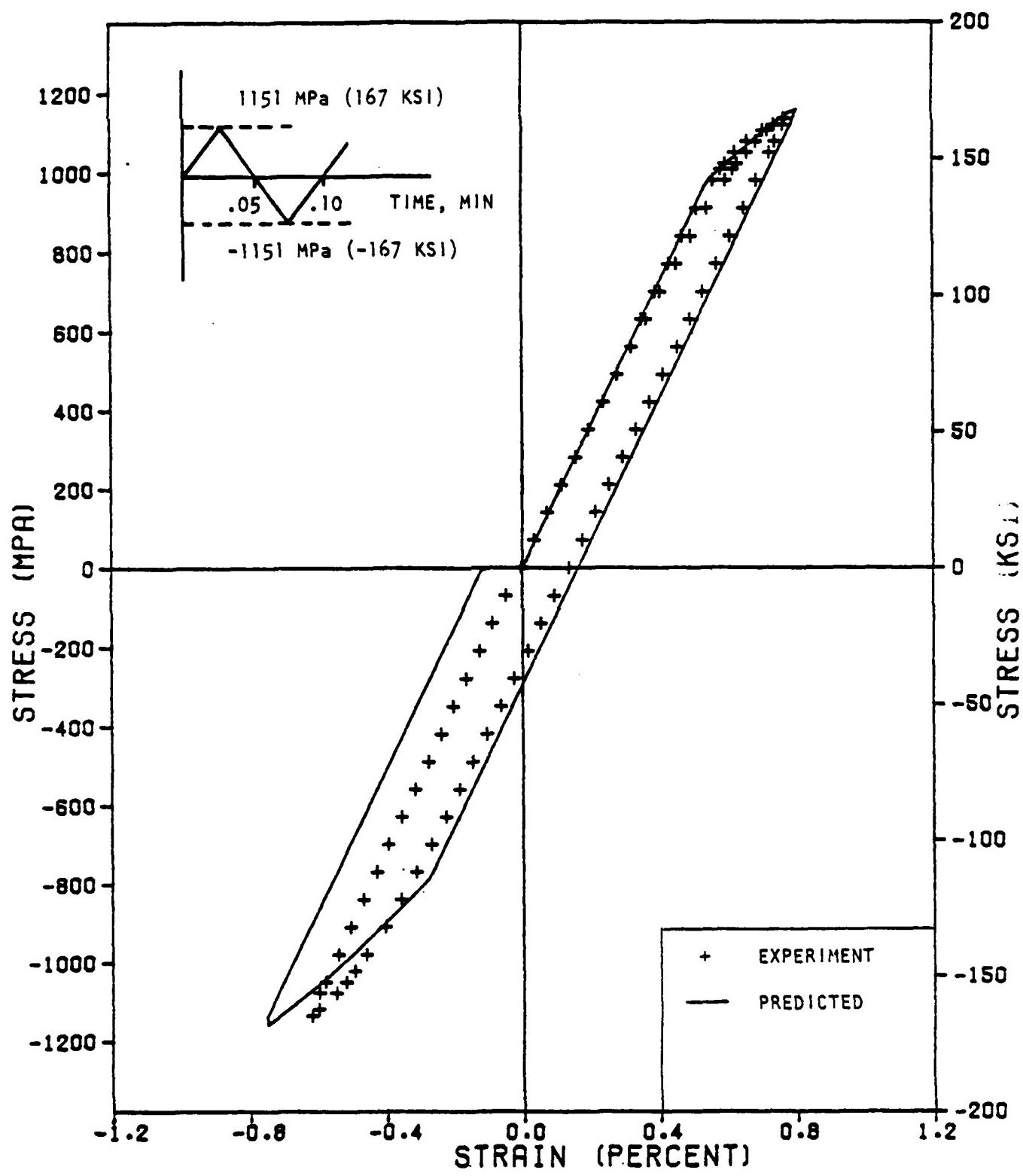


Figure 16. Experimental and predicted cyclic response using the state variable theory

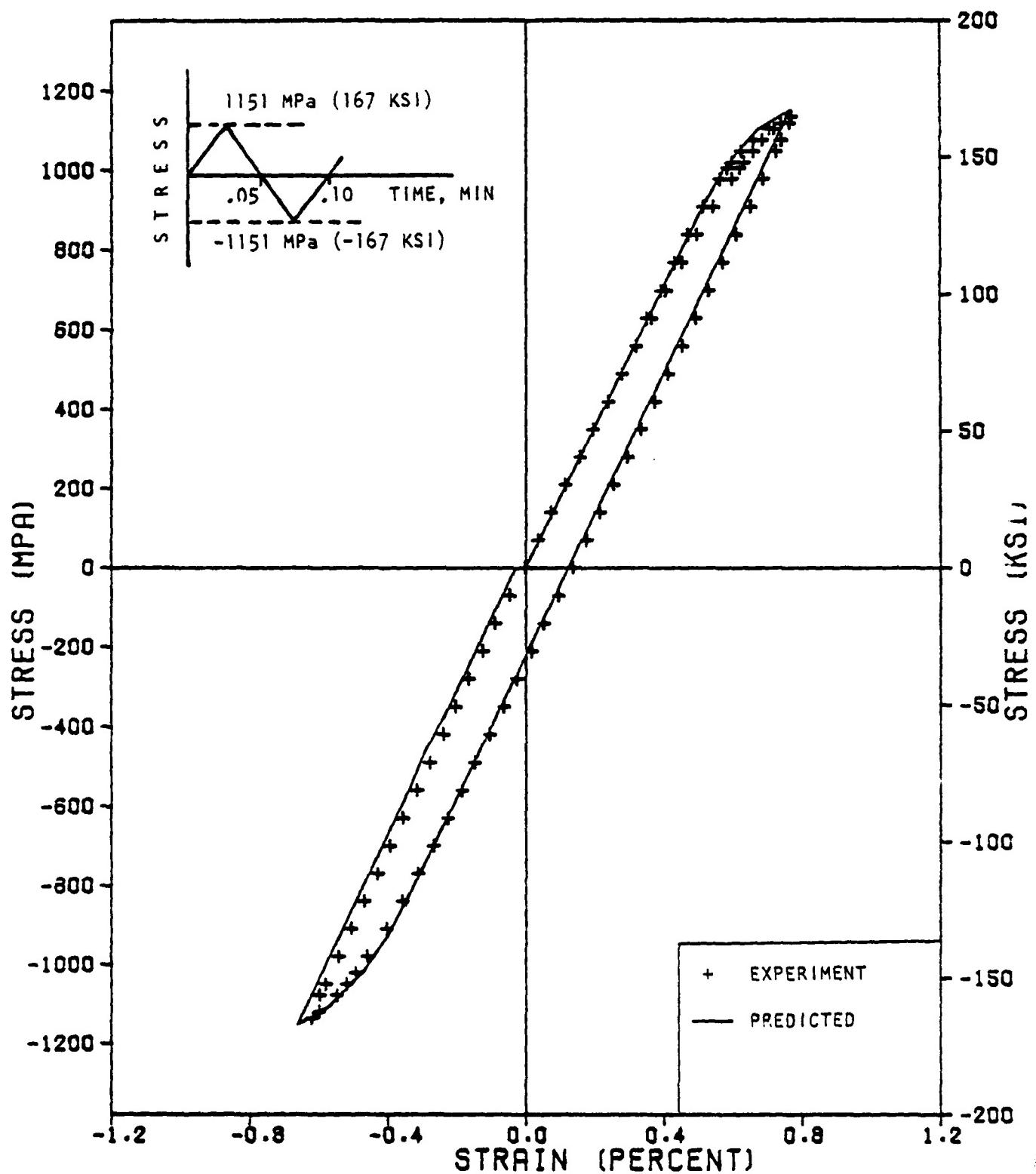


Figure 17. Experimental and predicted cyclic response using the non-linear creep theory

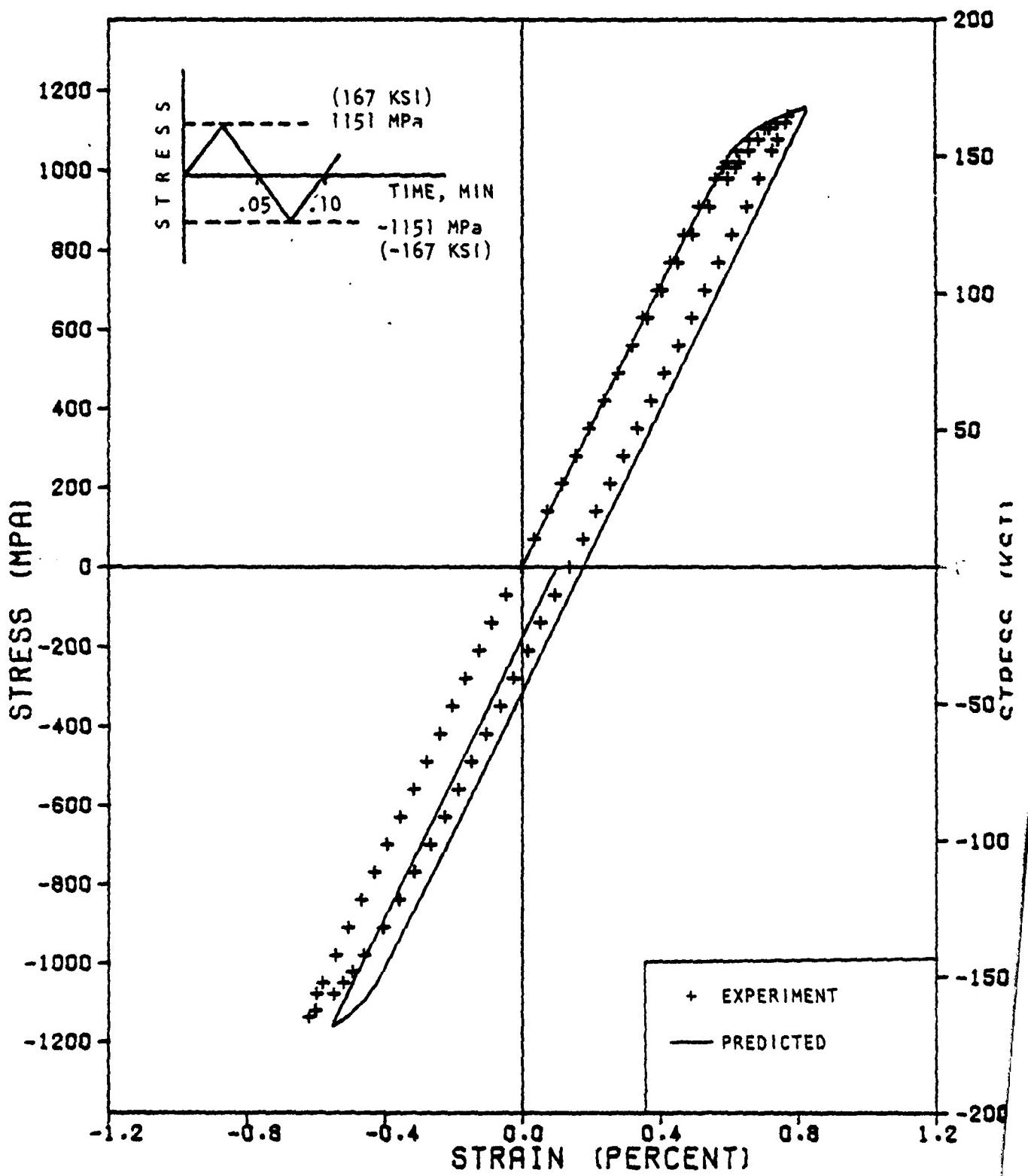


Figure 18. Experimental and predicted cyclic response using the isochronous theory

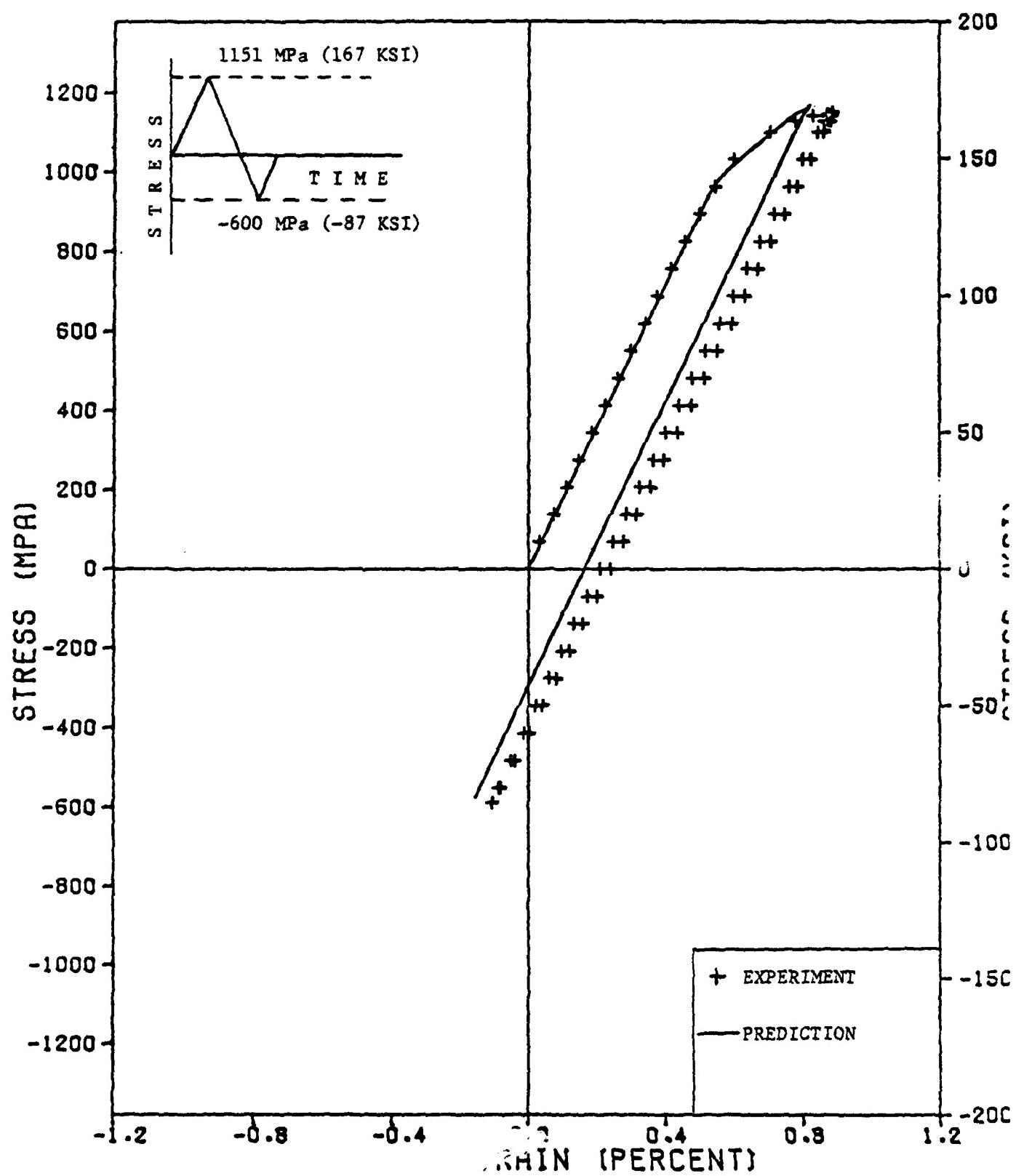


Figure 19. Experimental and predicted unbalanced cyclic response using the state variable theory

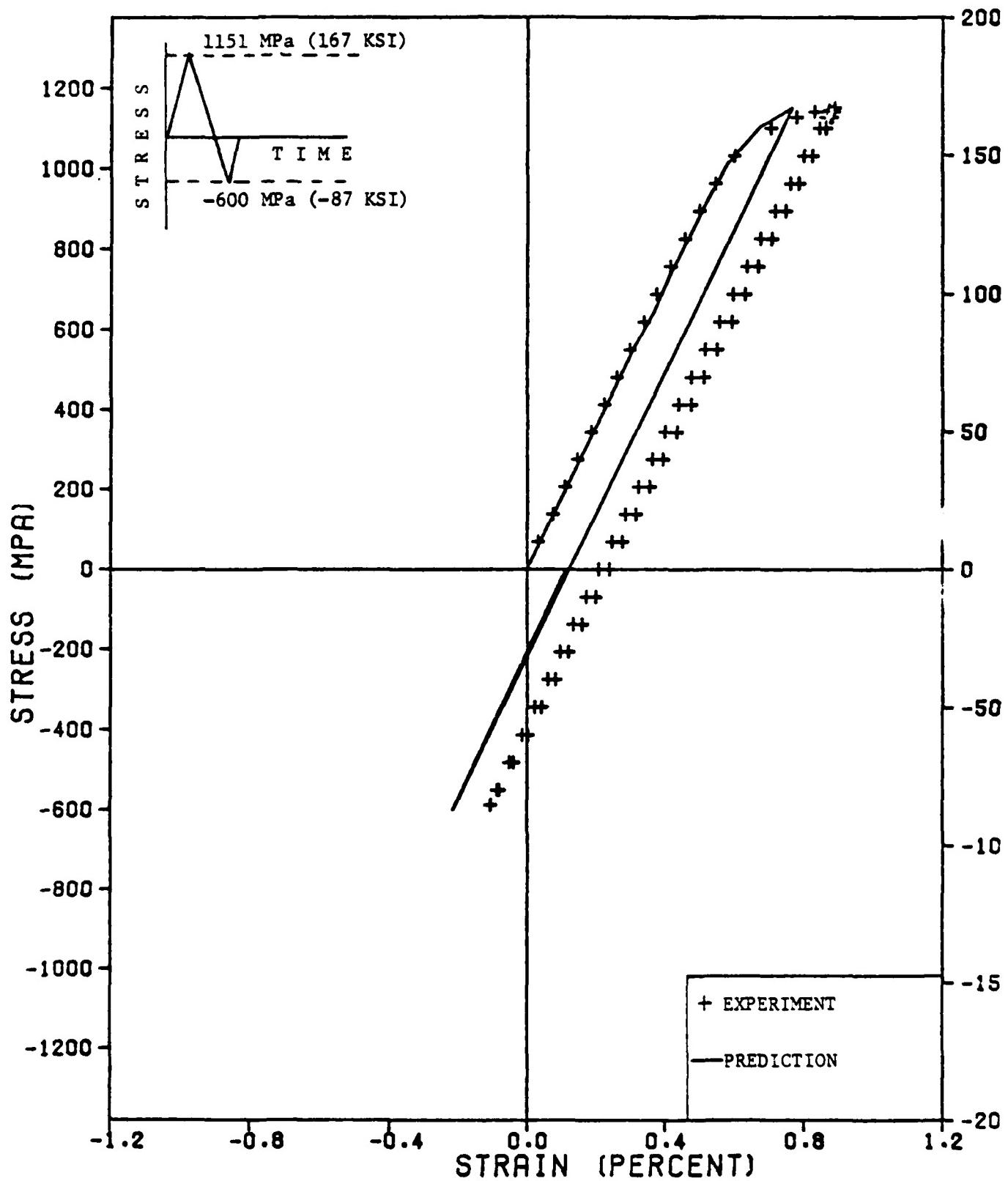


Figure 20. Experimental and predicted unbalanced cyclic response using the isochronous theory

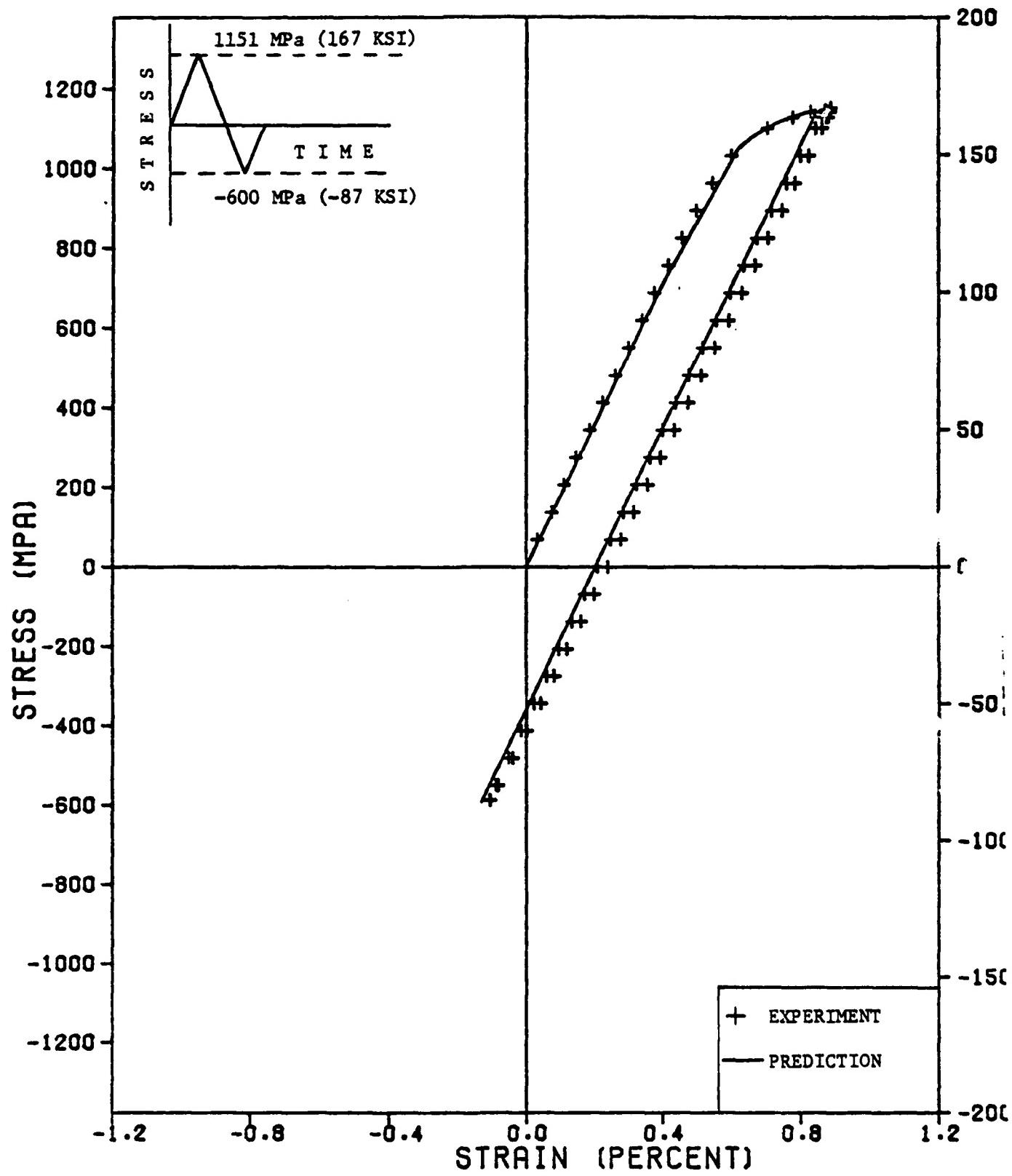


Figure 21. Experimental and predicted unbalanced cyclic response using the non-linear creep theory

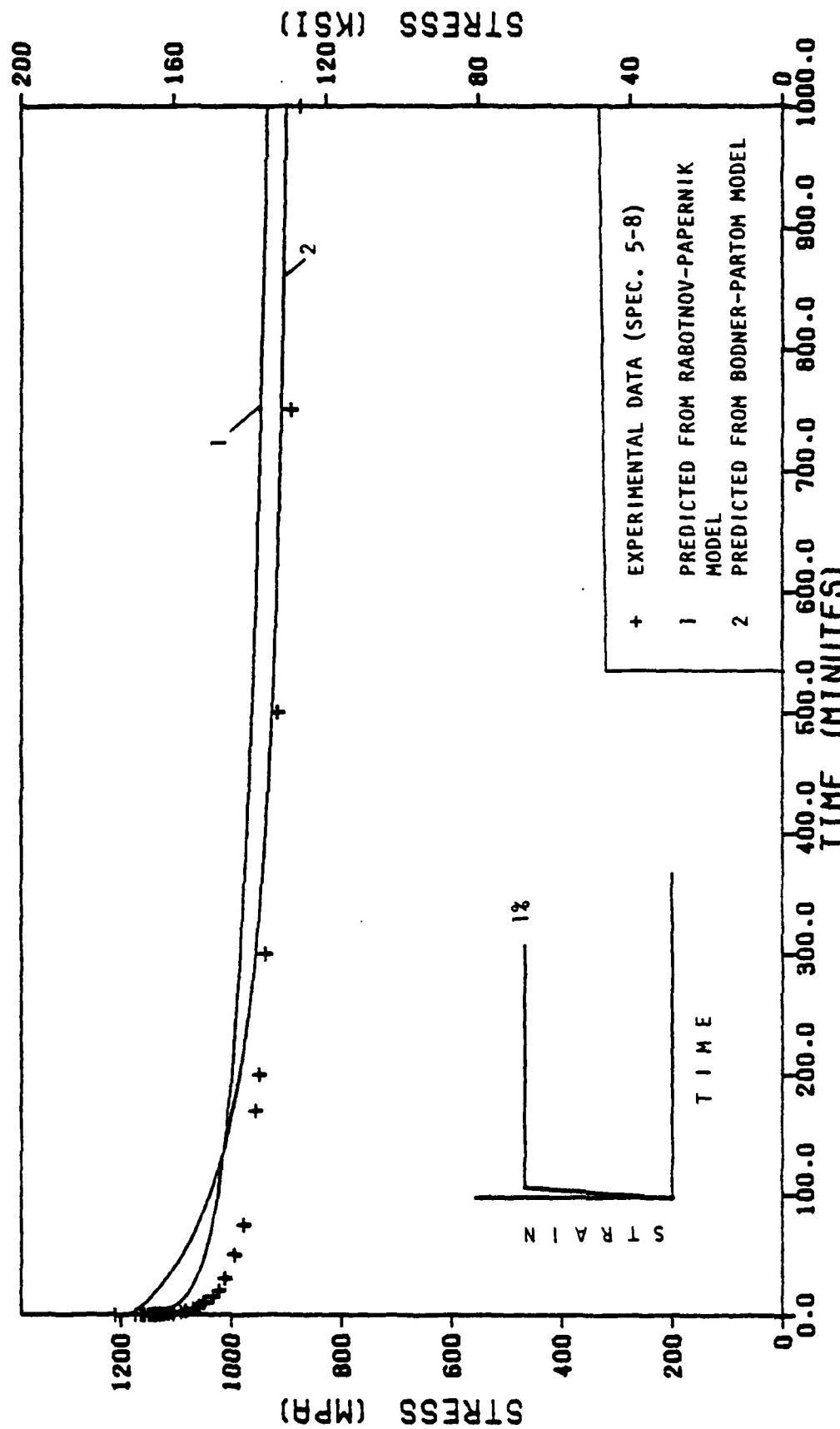


Figure 22. Experimental and predicted stress relaxation response using the state variable and isochronous theories

TABLE 4. PREDICTION OF CONSTANT STRAIN RESPONSE FROM THE STRESS RELAXATION DATA USING THE NONLINEAR CREEP MODEL.

Time, Min.	Stress MPA (KSI)	Total Strain Percent
2.0	1093 (158.5)	1.341
6.0	1063 (154.2)	1.317
10.0	1051 (152.5)	1.318
14.0	1033 (149.9)	1.305
18.0	1026 (148.8)	1.304
22.0	1021 (148.1)	1.305
26.0	1015 (147.3)	1.305
30.0	1010 (146.5)	1.305
34.0	1006 (145.9)	1.305
38.0	1002 (145.3)	1.305
42.0	998 (144.7)	1.305
46.0	993 (144.1)	1.305

SECTION X  
SUMMARY OF PREDICTIONS AND RESULTS

A review of the results suggests there are three major aspects of the response of René 95 that are not included in the above models. First, it is essential to include the compressive response characteristics. This involves developing a method of measuring high temperature compressive creep using two extensometers to compensate for bending as mentioned in Reference 1. Second, since two-thirds of the creep response prior to failure is in the tertiary creep domain, it is necessary to include tertiary creep effects in the models to predict plastic strains above 1.5%-2.0% in René 95. This could be important in finite element modeling of complex structural components. Third, the models should be modified to include cyclic history effects. This amounts to including tertiary creep and a damage measure in the mechanical constitutive equation. These topics must be addressed for a significant improvement in high temperature modeling of René 95.

Finally, it is appropriate to make a direct comparative study of the three models used in this investigation. This is shown in Table 5. It can be seen that no model can fully predict the entire list of response characteristics reviewed. Thus, one must choose the model that can best predict the response characteristics that are most important for a particular material or structural situation. In general, it appeared that it is easier to determine the material parameters in the Rabotnov-Papernik and Lafren-Stouffer models. However, the analysis of  $Z(W_p)$  presented in Part 2 is expected to lead to a direct method of determining the constants in the Bodner-Partom model. Conversely, the Bodner-Partom model is best suited for numerical computation. The use of the elapsed time,  $t - \tau$ , in the other models require integration on  $[0, t]$  for each choice of the current time  $t$ . This is a disadvantage if the stress or strain history is to be evaluated at a large number of time points such as in a finite element analysis of a structure.

TABLE 5. COMPARATIVE SUMMARY OF THE PREDICTED RESULTS

Response Property	Bodner-Partom	Rabotnov-Papernik	Model	Lafren-Stouffer
Tension	Shape of response curve could be improved as discussed in Part I. Model predicts strain-rate dependence.	Stress-strain response predicted from creep data. Model predicts correct shape of response curve but overpredicts rate dependence.	Stress-strain response predicted from creep data. Model predicts correct shape of response curve but does not include strain rate dependence.	
Creep	Creep predicted from both stress-strain and creep data. Model cannot predict tertiary creep in present form, but can be generalized to include damage effects.	Primary and secondary creep predicted to within data scatter. Model has ability to predict tertiary creep; but not included in the study.	Primary and secondary creep predicted to within scatter of data. Model cannot predict tertiary creep in present form, requires new creep equation.	
Cyclic Hardening	Can predict strain hardening effect. Bauchinger effect can be included with additional terms.	Cyclic histories can be included by updating coordinates at load reversal points in stress history. Model needs hardening term.	Cyclic histories can be predicted. Model cannot adequately predict strain hardening in its present form.	
Unloading	Nearly elastic unloading predicted as inherent part of model.	Linear unloading assumed in pseudo stress-strain space as part of model.	Elastic unloading assumed as part of model.	
Creep Recovery	Cannot predict creep recovery in present form; anelastic term can be developed.	Predicts creep and creep recovery using same logic.	Creep recovery excluded by choice of Marin-Pao creep model; another creep model is required.	
Determination of Material Parameters	Systematic method now exists for stress-strain terms. More work needed on creep terms.	Systematic method has been developed but ad hoc method more convenient for one material.	Coefficients determined directly from creep and stress-strain data. Ad hoc method required to determine viscoplastic parameter .	
Computation	Incremental time marching procedure applicable to equations. Computationally efficient model.	Elapsed time, $t - \tau$ , requires evaluating entire integral for each data point.	Elapsed time, $t - \tau$ , requires evaluating entire integral for each data point.	

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